



# Learning and Teaching with *Learning Trajectories*: Introduction

PROFESSIONAL DEVELOPMENT FACILITATOR GUIDE



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## Learning and Teaching with Learning Trajectories [LT]<sup>2</sup>: Website

[LT]<sup>2</sup> is a free web-based tool ([learningtrajectories.org](http://learningtrajectories.org)) for early childhood educators (including teachers and caregivers) to learn about how children think and learn about mathematics and how to teach mathematics to young children (birth to age 8/grade 3). [LT]<sup>2</sup> provides access to information about *Learning Trajectories* for mathematics, specifically in 17 distinct mathematics contents, and includes short video clips of children’s thinking along the *Learning Trajectories*. Hundreds of individual, small-group, and whole-class activity ideas to support children’s development along the mathematics trajectories are also provided.

### Using [LT]<sup>2</sup>

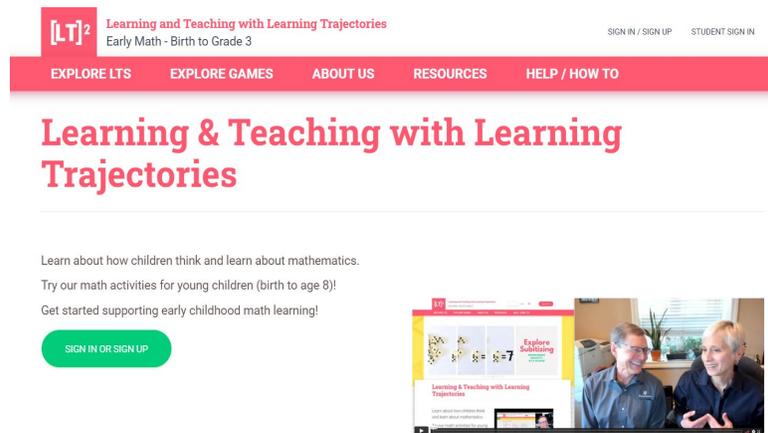
You must create a free account to use [LT]<sup>2</sup>. Visit the website <https://www.learningtrajectories.org/> watch the introductory videos on the homepage, and create an account if you are a new user. A complete manual on navigating the website and setting up an account is available in a user instructions document, “[Learning and Teaching with Learning Trajectories \[LT\]<sup>2</sup> – User Instructions](#)”.

Once logged in to the website, visit the **Explore LTs** page. The **Explore LTs** page includes 20 mathematics topics and their developmental progressions. You can expand or collapse the trajectories by clicking on the green arrow. When exploring the “Learning Trajectories” page, a learning trajectory level is highlighted when it corresponds with a selected age or a category (e.g., Number or Geometry). A level that is not highlighted can still be selected to access the content within that level. The highlighted sections merely give the levels coded for the specific age.

Start with any trajectory. Click on the heading for that trajectory for an overview of the trajectory, a navigable list of all of the levels in that trajectory, and an introductory video. You can access this video by clicking on the video camera icon next to each trajectory label. Navigate level-to-level within the trajectory by clicking on specific levels.

Within each level, note the descriptions and video(s) offered. Videos show the knowledge, behaviors, and reasoning children typically demonstrate when they are developing in this level. There are multiple videos for some levels, most often showing interview situations. Near the bottom of the page for each level will be activities (computer, small group, or whole class) to use when teaching children. These instructional activities are designed to help students reach the specific level you are on. The thumbnails with the play button indicate a video is available.

For additional information and details on navigating the website, visit the “[Learning and Teaching with Learning Trajectories \[LT\]<sup>2</sup> – User Instructions](#).”



# Learning Trajectories: Overview



[LT]<sup>2</sup> is, of course, all about *Learning Trajectories*. It is in the name! What are they? To begin, we know that children’s thinking follows natural developmental paths when learning mathematics. When educators working with young children understand these paths, and offer activities based on children’s progress along them, they build mathematical-learning environments that are developmentally appropriate and particularly effective. *Learning Trajectories* are a useful framework for understanding and supporting the development of children’s mathematical reasoning.

Our mathematics *Learning Trajectories* (Sarama & Clements, 2009) have three parts: 1) a mathematical

**goal**, 2) a **developmental path** along which children’s mathematics knowledge grows to reach that goal, and 3) a set of **instructional tasks**, or activities, for each level of children’s understanding along that path to help them become proficient in that level before moving on to the next level. Let’s examine each of these three parts.

**Goal.** The first part of a *Learning Trajectory* is the goal. Goals should include the big ideas of mathematics, such as “numbers can be used to tell us how many, describe order, and measure” (Sarama & Clements, 2008, p. 70) and “geometry can be used to understand and to represent the objects, directions, and locations in our world, and the relationship between them” (Sarama & Clements, 2008, p. 70).

**Developmental progression.** The second part of our *Learning Trajectory* consists of levels of thinking, each more sophisticated than the last, leading to achieving the mathematical goal (Sarama & Clements, 2009). That is, the developmental path describes a typical learning route children follow in developing understanding of and skill in a particular mathematics topic.

*Learning Trajectories* are important because young children’s ideas and their interpretations of situations are different from those of adults. Educators must interpret what the child is doing and thinking and attempt to see the situation from the child’s viewpoint. Knowledge of developmental paths enhances educators’ understanding of children’s thinking, helping educators assess children’s level of understanding and offer instructional activities at that level. Similarly, effective educators consider the instructional tasks from the child’s perspective.

**Instructional Tasks.** The third part of our *Learning Trajectories* consists of sets of instructional tasks or activities matched to each level of thinking in a developmental progression (Sarama & Clements, 2009). The tasks are designed to help children learn the ideas and practice the skills needed to master that level. Educators use instructional tasks to promote children’s growth from one level to the next

## [LT]<sup>2</sup> Professional Development: Overview

[LT]<sup>2</sup> professional development is professional development about *Learning Trajectories* that utilizes the [LT]<sup>2</sup> website. [LT]<sup>2</sup> professional development is intended to be given by a facilitator or a few facilitators to a group of participants (from a handful or more). Often, professional development is more effective with multiple facilitators, especially when the group of participants is large. For our purposes, **facilitator(s)** is defined as the individual(s) (e.g., instructional coaches, site leaders, professional development experts) leading [LT]<sup>2</sup> professional development. **Participants** are defined as those individuals receiving [LT]<sup>2</sup> professional development. Participants may come from a variety of backgrounds (e.g., teachers, caregivers, parents, paraprofessionals, administrators) and may be practicing or pre-service educators and caregivers. **Educators** are those who are teachers or caregivers of children birth through age 8/grade 3.

[LT]<sup>2</sup> professional development is highly interactive. Trainings are structured so that facilitators show participants how to navigate the website; thus a computer, projector, and internet must be available. The facilitators will also show videos from the website to the participants, therefore, a sound system must be available. Participants will engage in many activities available on the website—thus the facilitators will have to carefully prepare materials. Participants can expect to leave an [LT]<sup>2</sup> professional development with an understanding of content, website navigation, and experience with activities they can use with children in one-on-one, small group, and whole class settings.

The rest of this document outlines in more depth the demands of an [LT]<sup>2</sup> professional development and plans for facilitating the Introduction.

### Learning Trajectory-based Teaching and Professional Development

*Learning Trajectory*-based teaching requires knowledge of each of the three components of the *Learning Trajectories*. Therefore, professional development must ensure that training addresses each of the three components of the *Learning Trajectories*.

- To understand the **goals**, educators/participants must understand core mathematics concepts and procedures for each topic. For example, in professional development, participants might discuss the system of verbal counting based on cycling through 10 digits and the concept of place value.
- To understand the **developmental progressions**, educators/participants must understand the levels of thinking from birth through age 8/grade 3/the goals for each particular mathematics topic. For example, in professional development, participants might view video segments that illustrate each level and discuss the mental “actions on objects” that constitute the cognitive components of each level.
- To understand the **instructional tasks**, educators/participants must \*\*\*. For example, in professional development, participants might study the tasks or activities in the curriculum, practice enacting them with each other, and also view, analyze, and discuss video of the enactments in classrooms.

Further, educators need help learning how to use the *Learning Trajectories* as a basis for formative assessment, a key to high-quality teaching. Educators need to discuss and practice how to interpret children’s thinking and select appropriate instructional tasks for the class (e.g., compacting the curriculum if most children can learn it at a faster pace) and for individuals.

## [LT]<sup>2</sup> Professional Development Goals

- What *Learning Trajectories* are in general and understand the three components of a *Learning Trajectory*.
- Specific *Learning Trajectories* for major mathematics topics for children birth through Grade 3.
- How the *Learning Trajectories* align with teaching and learning standards (e.g., Teaching Strategies Gold).
- How to use *Learning Trajectories* for formative assessment.
- How to support individual mathematical development using *Learning Trajectories*.
- How to use *Learning Trajectory*-based computer games, including how to set up learning centers with computers and the use of the [LT]<sup>2</sup> management system.
- How to enact small-group and whole-class activities aligned with *Learning Trajectories*.

From [LT]<sup>2</sup> professional development, participants will learn:

## [LT]<sup>2</sup> Professional Development Facilitator Guides

The [LT]<sup>2</sup> Professional Development Facilitator Guides (abbreviated to just Facilitator Guides henceforth) are intended for individuals who are responsible for facilitating professional learning opportunities for educators and caregivers, of children from birth to age 8/grade 3, in mathematics education.

Facilitator Guides are not meant to be rigid instructions. They are to be used as practical resources and purposeful references for facilitators who have a sound understanding of early mathematics *Learning Trajectories*. Professional development will be more effective, and facilitators will be more comfortable, if they bring their own unique style and experiences.

This introductory Facilitator Guide provides professional development facilitators with notes on general preparations and tips for introducing *Learning Trajectories* in general and the [LT]<sup>2</sup> tool specifically to participants. Other, content-specific Facilitator Guides provide professional development that builds on the foundational understanding outlined in this Introduction. All participants should receive the Introduction before any content-specific [LT]<sup>2</sup> professional development.

This Facilitator Guide outlines one 30- to 60-minute introductory professional development. All participants should receive the Introduction before any content-specific [LT]<sup>2</sup> professional development. Content-specific Facilitator Guides are organized by mathematical topics and age/grade levels. Each content-specific Facilitator Guide includes ways to support participants' learning of learning trajectories in a specific mathematical content.

## Considerations for Professional Development

- Participants will bring diverse experiences, from those experienced working with young children to those who are completing their first year of teaching or preparing for a teaching career. There may be lead teachers, assistant teachers, and paraprofessionals, even students and caregivers or parents. It is important to find ways to engage all participants and use their strengths to enhance the professional development.
- Participants might have varying degrees of comfort and knowledge with mathematics. The participants might also have varying degrees of comfort and knowledge with the ideas and documents and resources that describe them. The more familiar you are with mathematics teaching and learning and the [LT]<sup>2</sup> ideas and documents and resources, the better able you will be to help participants connect their own experiences and learning.
- Throughout the session, look for opportunities to highlight the expertise of experienced participants, and encourage them to share their knowledge with others. For example, you may listen for particularly insightful comments during small group discussions and ask the participant to share with the whole group.
- A variety of techniques, representing the different ways that people learn, have been incorporated into the sessions. They include mini-lectures, videos and information on [LT]<sup>2</sup>, “think-pair-share” interactions, recording on chart paper, and large and small group work. Feel free to adapt these activities, taking into consideration the size of the group, the venue, and the experience levels of the participants. Consider the expertise of participants (e.g., including someone experienced working with a child with a disability during discussions about differentiating activities for children with special needs).
- Do not feel the need to do every activity presented in the Facilitator Guides. They are merely illustrations. For the sake of time, or interest, use only those you believe will help the participants and meet the objectives of your specific session.

## Setting Up, Organizing, Materials, Technology

For many facilitators, these suggestions will be well-known, but perhaps useful reminders. Prepare a welcome sign on chart paper. Arrange participant handbooks (if applicable), markers, math manipulatives, chart paper, post-its, and pens in a way that makes sense for the space. You may want to add additional elements to create a welcoming space (e.g., music, children’s books on transitions, chart paper with agenda, etc.). Create a chart titled “Parking Lot” for questions that arise during the session which are not relevant to the topic.

### Room organization and materials/technology

Prior to the training facilitators should familiarize themselves with the manipulatives to be used by participants. Facilitators should be comfortable identifying appropriate manipulatives when participants have questions and should be able to demonstrate appropriate use of materials as needed.

Let participants know that they will be working in small groups. The groups should be made up of about 4-5 participants.

Ask participants to form small groups.

You may wish to have each small group focus on the needs of a specific group of children:

- Children whose native language is not English
- Children with an Individualized Education Plan (IEP)
- Children who need targeted support

Explain that we are forming groups that reflect the diverse needs of children in classrooms and assigning roles to participants so that the needs of each group are considered and included in discussions and planning.

Describe each category to the participants and recommend that participants take a role with which they have expertise, experience, or interest.

**All participants must have the following during training:**

- Activities (from [LT]<sup>2</sup>) identified for practice in the *Learning Trajectory*-specific Guides.
  - Participants must have a copy of each pdf document of the activities as you have a projector and so forth to view the videos with the whole group *or*
  - Participants must be on computers (paired up is fine) for downloading the documents and viewing the videos.
- Access to manipulatives.

**Facilitators are responsible for ensuring materials and technology components are ready.**

## Suggestions for Co-Facilitation

In the event that you are co-facilitating your session please keep in mind the following practices. Co-facilitation allows one person to present while the other observes and supports their partner. Partners should divide the material in a way that lets them capitalize on individual strengths and have their own moments to lead discussion. Remember that you are modeling co-teaching practices for the participants.

### **Before the Training**

- Schedule time for planning together.
- Discuss each other's style of planning and facilitating.
- Take time to discuss your views about the training materials.
- Especially examine areas of disagreement.
- Discuss any concerns about potential challenges that participants may present.
- Find out whether and when it is okay to interrupt.
- Decide how to keep track of time.
- Plan ways to give signals to one another
- Divide facilitation of activities fairly.
- Share responsibility equally in preparing and bringing training materials and resources.
- Agree to arrive at the training site in time to set up and check-in before the training begins.

### *During the Training*

- Keep communicating with each other throughout the training.
- During activities that don't require constant attention, check-in with one another
- Include your co-facilitator even when you are leading an exercise or discussion, by asking, for example: "Do you have anything to add?"
- Two facilitators can manage a group better than one. The second person can help gauge participants' reactions and notice whether people seem to be on-track.
- Co-facilitators can also help hand out materials, assist in monitoring discussions and/or coach participants in groups.
- Co-facilitators can monitor and handle problems with the physical environment, latecomers, phone calls, audio-visuals, and other logistical matters.
- Use lots of eye contact.

## Before Participants Arrive

- Set up the room as you would like it to be used once participants arrive.
- Check technology—computer projection, internet, volume on sound system.
- Log in to the [LT]<sup>2</sup> website.

## Once Participants Arrive

### Welcome

- Welcome the group warmly and introduce yourself.

### Seating

- Reorganize participants to desired groupings.

### Getting Started

- Remind participants of the time and topic for the professional development.
- Introduce the Parking Lot and procedures for use.

## Training Agenda

### Objectives

- Go over objectives for [LT]<sup>2</sup> Introduction professional development with participants:
  - (1) Understand the three components of a *Learning Trajectory* and gain insight into learning trajectory-based teaching and learning.
  - (2) Be able to navigate the [LT]<sup>2</sup> website.
  - (3) Address comments, questions, and concerns from the Parking Lot.

## Starting the First Session: Introducing Learning Trajectories and the [LT]<sup>2</sup> Tool

<b>Objectives</b> <ul style="list-style-type: none"> <li>• Understand the importance of early mathematics in children’s development.</li> <li>• Understand the components of a <i>Learning Trajectory</i>.</li> <li>• Be able to navigate the [LT]<sup>2</sup> website.</li> </ul>	
<b>Facilitation Materials</b>	<b>Directions</b>
<p>[LT]<sup>2</sup> User Instructions linked <a href="#">here</a>.</p> <p>Access to the Internet</p> <p><i>Learning Trajectories</i> overview document linked <a href="#">here</a>.</p>	<p><b><u>Introduce Learning Trajectories</u></b></p> <ol style="list-style-type: none"> <li>1. Ask participants what they have heard about <i>Learning Trajectories</i>.</li> <li>2. Play the introduction to <i>Learning Trajectories</i>: <a href="https://www.learningtrajectories.org/lt-resources/what-are-learning-trajectories">https://www.learningtrajectories.org/lt-resources/what-are-learning-trajectories</a></li> </ol> <p><b><u>Discuss:</u></b></p> <p>What are the primary components of a <i>Learning Trajectory</i>?</p> <p>The goal, the developmental levels, and the instructional activities to help children get to the next level to meet the goal.</p> <p>How do <i>Learning Trajectories</i> help guide students toward the mathematical goal in these levels?</p> <p>The instruction is key. A learning trajectory approach is not about targeting a child at a level and periodically reassessing. The instructional activities are intended to be the “just right” instruction children need.</p> <p><b><u>Introduce Site Navigation</u></b></p> <ol style="list-style-type: none"> <li>1. Ask participants to watch the two videos at the bottom of the home page to create an account and learn about navigating the site. <a href="https://www.learningtrajectories.org/early-math/birth-to-grade-3">https://www.learningtrajectories.org/early-math/birth-to-grade-3</a></li> </ol> <p>How can the <i>Learning Trajectories</i> help you in your planning and teaching?</p> <p>Answers will vary, but participants should know that this is a developmental perspective, that learning more about how children learn mathematics will enable them to better see children’s strengths, meet children where they are, and differentiate appropriately to help all children meet the goals.</p> <p>How does filtering the age or category change the highlighted content on the page?</p> <p>Explore [LT]<sup>2</sup> with users with the guiding questions to the right and using the background information below.</p>
<b>Background Information</b>	
<p>When exploring the developmental progression page, remember a level is highlighted when a specific age or a category is selected, such as number or geometry. These categories, or strands are also color-coded – with Number having a yellow background; Operations topics are orange; Geometry is pink; Measurement is green. A participant can select a level that is not highlighted and still access the content within that level. The highlighted sections merely give the levels that are coded for the specific age.</p>	

When starting with the website for the first time, it is useful to explore the developmental progressions, starting with the early levels and moving up to more advanced levels.

Videos show children's development. Note that there are multiple videos for some levels and that these are most often interview situations.

Instructional activities here are designed to help students reach that level. The thumbnails with the play button indicate a video is available.

Talk with participants about positive adaptations and lethal mutations when adjusting for their classroom contexts.

Computer screen icons appear throughout the *Guides* directing facilitators to play videos from the [LT]<sup>2</sup> website. Facilitators should familiarize themselves with the website to have an in-depth working knowledge of the resource. Facilitators should make sure this website is loaded on computers and easily accessible for presentation when the time comes.

## References

- Clements, D. H., & Sarama, J. (2020). *Learning and Teaching Early Math: The Learning Trajectories Approach (3rd ed.)*. Routledge. <https://doi.org/10.4324/9781003083528>
- Sarama, J., & Clements, D. H. (2008). Mathematics in early childhood. In O. N. Saracho & B. Spodek (Eds). *Contemporary perspectives on mathematics in early childhood education* (pp. 67 -94). Charlotte, NC: Information Age Publishing, Inc.
- Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York, NY: Routledge.

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