Trajectory Level	Standards	
Adding / Subtracting		
Adding & Subtracting: Arithmetic Senser:Foundations	Very young children are sensitive to combining or separating perceptual groups. An infant may observe, point, or make sounds while someone else introduces a quantity of objects. They will notice the effects of increasing or decreasing small collections by one item and may also be sensitive to the results of combining larger groups.	
Adding & Subtracting: Counting Strategies +/-	Finds sums for joining ("You had 8 apples and get 3 more …") and part–part–whole ("6 girls and 5 boys …") problems with finger patterns and/or by counting on.	
	1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = � – 3, 6 + 6 = �.
	K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
	1.OA.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.2
	K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
Adding & Subtracting: Deriver +/-	Uses flexible strategies and derived combinations (e.g., "7 + 7 is 14, so 7 + 8 is 15) to solve all types of problems. Includes "Break Apart to Make Ten" (BAMT; explained in Chapter 6). Can simultaneously think of 3 numbers within a sum, and can move part of a number to another, aware of the increase in one and the decrease in another. May solve simple cases of multidigit addition (sometimes subtraction) by counting by tens and/or ones.	

Trajectory Level	Standards	
	1.OA.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
	2.OA.2	Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.
	1.NBT.4	Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
Adding & Subtracting: Find Change +/-	Finds the missing addend (e.g., 5 + _ = 7 or 9 - _ = 3) to solve Join and Separate, Change Unknown problems by adding on or taking away objects. Compares by matching in simple situations.	
	K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

Trajectory Level	Standards	
	K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
Adding & Subtracting: Find Result +/-	Finds sums for Join, Result Unknown problems ("You had 3 apples and get 3 more, how many do you have in all?") and part–part–whole ("There are 6 girls and 5 boys on the playground, how many children were there in all?") problems by direct modeling, counting all, with objects. Solves take-away problems by separating with objects.	
	K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
Adding & Subtracting: Make it N	Adds on objects to "make one number into another," without needing to count from 1. Does not (necessarily) represent how many were added (this is not a requirement of this intermediate-difficulty problem type)	
	K.OA.2	Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
Adding & Subtracting: Numbers-in-Numbers +/-	Recognizes when a number is part of a whole and can keep the part and whole in mind simultaneously; solves "Start Unknown (e.g., _ + 4 = 9) problems with counting strategies.	
	2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 1

Trajectory Level	Standards	
	1.OA.3	Apply properties of operations as strategies to add and subtract.3 Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)
Adding & Subtracting: Part-Whole +/-	Has initial part–whole understanding and can solve all previous problem types using flexible strategies. May use some known combinations, such as 5 + 5 is 10. Sometimes can do "Start Unknown (e.g., _ + 6 = 11), but only by trial and error.	
	1.OA.4	Understand subtraction as an unknown-addend problem. For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.
	K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
Adding & Subtracting: Preverbal	Adds and subtracts very small collections (totals up to three), often making a collection rather than answering verbally.	
Adding & Subtracting: Problem Solver +/-	Solves all types of problems, with flexible strategies and known combinations. Multidigit may be solved by incrementing tens and ones by counting (latter not used for Join, Change Unknown).	

	2.NBT.7	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.
	2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 1
	2.NBT.6	Add up to four two-digit numbers using strategies based on place value and properties of operations.
	2.OA.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. 1
Adding & Subtracting: Small Number +/-	Finds sums for Join, Result Unknown and Separate, Result Unknown problems with totals up to5 by “counting all” with objects.	
	K.OA.1	Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.

Trajectory Level	Standards
Angle and Turn	
Angle and Turn measurement: Angle and Turn Senser: Foundations	Infants are sensitive to angles-as-turning, both turning objects and their own body. See more at the first three levels of the Spatial Orientation and the first level of Spatial Visualization.
Angle and Turn measurement: Angle Matcher	Matches angles concretely. Explicitly recognizes parallels from non-parallels in specific contexts (Mitchelmore, 1992). Sorts angles into “smaller” or “larger” (but may be misled by irrelevant features, such as length of line segments).
Angle and Turn measurement: Angle Measurer	Understands angle and angle measure in both primary aspects, and can represent multiple contexts in terms of the standard, generalizable concepts and procedures of angle and angle measure (e.g., two rays, the common endpoint, rotation of one ray to the other around that endpoint, and measure of that rotation).
Angle and Turn measurement: Angle Size Comparer	Differentiates angle and angle size from shapes and contexts, and compares angle sizes. Recognizes right angles, and then equal angles of other measures, in different orientations. Compares simple turns. (Note that, without instruction, this and higher levels may not be achieved even by the end of the elementary grades.)

Trajectory Level	Standards
Angle and Turn measurement: Implicit Angle User	Uses angles and, at least implicitly, some angle measure concepts, such as parallelism and perpendicularity—in physical alignment tasks, construction with blocks, or other everyday contexts. May identify corresponding angles of a pair of congruent triangles using physical models. Uses the word “angle” or other descriptive vocabulary to describe some of these situations.
Angle and Turn measurement: Intuitive Angle Builder	Intuitively uses some angle measure notions in everyday settings, such as building with blocks, solving puzzles, and walking.

Trajectory Level	Standards	
Area		
Area: Area Quantity Recognizer	Perceives the amount of two-dimensional space and can make intuitive comparisons. However, when asked to compare, may compare lengths more than areas because lengths are salient and familiar to them (e.g., compare one side of one piece of paper to the side of another) or make estimates based on a “length plus (not times) width” intuition. However, may compare areas correctly if the task suggestions superposition (putting one on top of the other). Asked to partition a space into squares or copy an image of a rectangle partitioned into an array (rows and columns), may simply draw squares (usually!) inside the rectangle or other types of shapes or short paths on or around the rectangle.	
Area: Area Row and Column Structurer	Decomposes and recomposes partial units to make whole units. For example, draws rows as rows making parallel horizontal lines and so forth. Begins conserving area and reasons about additive composition of areas (e.g., how regions that look different can have the same area measure) and recognizes the need for space-filling in most contexts.	
	3.MD.7.c	Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
	3.MD.7.b	Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Trajectory Level	Standards
	3.MD.7.a Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
	3.MD.7 Relate area to the operations of multiplication and addition.
	3.MD.7.d Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.
Area: Area Senser: Foundations	Even children in their first year are sensitive to area. However, may not explicitly recognize area as an attribute (separate from general size, such as “small” and “big”) for some time. If asked to fill in a rectangle, preschoolers may just draw approximations of circles (Mulligan, Prescott, Mitchelmore, & Outhred, 2005). Uses side matching strategies in comparing areas.
Area: Area Unit Relater and Repeater	Counts individual units, often trying to use the structure of rows. To cover a region with physical units, repeats (iterates) an individual unit. Draws a complete covering based on an intuitive notion of rows and columns, making equal sized units, but often draws them one at a time. That is, draws individual, mainly equal-sized units that are lined up but may not see groups of units making up individual rows or columns. Relates the size and number of units to cover a region, recognizing that differently sized units will result in different measures and that the larger the unit, the fewer will be needed. Compares areas by accurately counting units in each and comparing the resulting measures.

Trajectory Level	Standards	
	3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement.
	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
	3.MD.5.a	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
	3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement.
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	3.MD.5.a	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Trajectory Level	Standards
	3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). **2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
Area: Array Structurer	With linear measures or other similar indications of the two dimensions, multiplicatively iterates rows or columns to determine the area. Does not need to draw in the array to do so. Has an abstract understanding of the rectangular area formula. Understands and justifies that differently-shaped regions can have the same areas. Compares regions with transitive reasoning (e.g., A is greater than B, B is greater than C, so I know A is greater than C).
	3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **3.MD.7.a** Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
	3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **3.MD.7.b** Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
	3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **3.MD.7.d** Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Trajectory Level	Standards
	3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **3.MD.7.c** Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
	3.MD.5.b A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units. **3.MD.7** Relate area to the operations of multiplication and addition.
Area: Complete Coverer and Counter	Draws a complete covering of a specific region without gaps or overlaps and in approximations of rows. When provided with more than the total number of physical tiles needed, can build a region of specified area (e.g., build a rectangle with an area of 12 from a pile of 20 tiles).
	2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
Area: Initial Composite Structurer	Identifies a square unit as both a unit and a component of a larger unit of units (a row, column, or group) and uses those structures in counting or drawing. However, needs figural support to structure the space themselves (this may include physical motions of some of the tiles or drawing some collections of units rather than from using the dimensions). At this level, usually does not coordinate the width and height and in measuring, may not use the dimensions of the rectangle to constrain the unit size. Makes reasonable estimates of areas.
	3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement. **2.OA.4** Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Trajectory Level	Standards	
	3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement.
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	3.MD.5.a	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.
	3.MD.5.a	A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
	2.G.2	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
	3.MD.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
	2.OA.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Trajectory Level	Standards
Area: Physical Coverer and Counter	Prompted to measure, attempts to cover a rectangular space with physical tiles. However, doesn't organize or structure the 2D space without considerable perceptual support, such as a grid that outlines each individual unit. In drawing (or imagining and pointing to count squares as units of area), represents only certain aspects of that structure, such as approximately rectangular shapes next to one another. Makes comparison areas based on simple, direct comparisons (e.g., a child places one sheet of paper over another piece of paper to select the sheet that covers more space).

Trajectory Level	Standards
Classification and Data Analysis	
Classification: Consistent, Flexible Sorter	Sorts consistently by a single attribute and re-classifies by different attributes. Sorts consistently and exhaustively by an attribute, given or created, and uses the terms "some" and "all."
	K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.
Classification: Data Aggregater	Classifies objects that may be perceptually different by more abstract attributes such as function or conceptual attributes. Focuses on features of the data set as a whole. Uses to describe relative frequency and density (shape), and location (centers). Begins to understand the concepts of expectation (averages and probabilities) and variation ("spread" of values). Views ranges in data or view the mode (the number or range of numbers that occurs most frequently). Eventually, can focus on features of the data set as a whole, including the relative frequencies, density ("shape"), and location (centers, such as the mean).
	1.MD.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Classification: Data Case Viewer	Associates a value with an individual case. Uses numeric data to identify largest/smallest cases. Before this level, children may be "pointers" in which data records point to the entire event ("We talked about favorite colors"). They use it like string tied around a finger, to remember that they did something.
	K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

Learning Trajectories: Alignment to CCSS

Trajectory Level	Standards	
Classification: Data Classifier	Data Treats cases with similar values as the same. Uses to compare category frequencies (most and least popular case-types). Visually compares two graphs.	
	1.MD.4	Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.
Classification: Data Representer	Shows an appreciation of the “center” of graphs and for their variation or spread. Compares graphs of data sets of the same size accurately.	
	2.MD.10	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put together, take-apart, and compare problems using information presented in a bar graph
Classification: Foundations: Similarity Intuiter	Intuitively recognizes objects or situations as similar in some way (objects to suck or not, 2 weeks). Places objects together that are different (6 months) and then alike (12 months).	
Classification: Hierarchical Classifier	Classifies categories and subcategories using hierarchical inclusion. Conscientiously classifies according to multiple attributes, naming and relating the attributes, understanding that objects could belong to more than one group.	
Classification: Multiple Attribute Classifier	Classifies objects by multiple attributes in a single sort.	

Trajectory Level	Standards
Classification: Similar/Dissimilar Maker	By 18 months, forms sets in which objects in each set are identical and objects in the other sets are different, and by 2 years, intuitively forms groups with objects that are similar on some attributes (may be mixed and inconsistent), but not necessarily identical.
Classification: Simple Sorter	Follows verbal rules for sorting scaffolded by an adult. (These may be made with shifting criteria; nevertheless, they play an essential role in number, through the unitizing process.) Can “fix” a simple sort with mistakes.
Classification: Sorter by Similar Attributes	Sorts objects according to an explicit attribute (although still may decide to switch attributes during the sorting). The end result may appear to reflect adult categorizations, but often has a different basis, such as general resemblance.

Trajectory Level	Standards
Comparing Number	
Comparing Number: Benchmarks Estimator	Counts a portion of the to-be-estimated collection and uses that as a benchmark from which an estimate is made, intuitively or using repeated addition or multiplication. Scanning can be linked to recalled benchmarks.
Comparing Number: Comparison Senser: Foundations	From the first months of life, children are sensitive to a change in the number, either of a change of very small collections, such as 1 vs. 2, or large changes in larger collection, such as double the number. Therefore, we know infants have an unconscious, innate sensitivity to such simple equivalence comparisons.
Comparing Number: Composition Estimator	Decomposes or partitions the to-be-estimated set into convenient subset sizes, then recomposes the numerosity. Initially, this is done with regular arrangements using repeated addition or multiplication. Later, the process can be done with irregular arrangements and children more consistently use multiplication skills to recompose.
Comparing Number: Counting Comparer (10)	Compares with counting, even when larger collection's objects are smaller, up to 10.
Comparing Number: Counting Comparer (5)	Compares with counting, even when larger collection's objects are smaller. Later, figures out how many more or less.

Trajectory Level	Standards
	K.MD.3 Classify objects into given categories; count the numbers of objects in each category and sort the categories by count. **K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies. **K.CC.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
Comparing Number: Counting Comparer (Same Size)	Accurately compares via counting, but only when objects are about the same size and groups are small (up to about 5). Not always accurate when the larger collection's objects are smaller in size than the objects in the smaller collection. Accurately counts two equal collections, but, when asked, says the collection of larger blocks has more.
Comparing Number: Early Comparer of Dissimilar Items	Matches small, equal collections consisting of different items, showing that they are the same number.
Comparing Number: Early Comparer of Similar Items	Compares collections of 1 to 4 items verbally or nonverbally ("just by looking"). The items must be the same. May compare the small collections using number words "two" and "three" (approximately age 3; 2), and "three" and others (age 3; 6). Some do this even before they can accurately by using recognition of number/subitizing for these quantities. May transfer an ordering relation from one pair of collections to another.

Trajectory Level	Standards	
Comparing Number: First-Second Ordinal Counter	Identifies the "first" and often "second" objects in a sequence.	
Comparing Number: Many-to-One Corresponder	Recognizes that two very small collections have the "same number" by intuitively making a correspondence between the items in each collection. At this level, in certain situations, children may also put objects, words, or actions in one-to-one or many-to-one correspondence or a mixture.	
Comparing Number: Matching Comparer	Compares groups of 1–6 by matching.	
Comparing Number: Mental Number Line to 10	Uses internal images and knowledge of number relationships to determine relative size and position.	
	K.CC.7	Compare two numbers between 1 and 10 presented as written numerals.
Comparing Number: Mental Number Line to 100	Uses knowledge of number relationships and mental images, including how ones can be embedded in tens, to determine relative size and position.	
	1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
	1.NBT.2.c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

Trajectory Level	Standards	
	1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
	1.NBT.2.c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
Comparing Number: Mental Number Line to 1000	Uses internal images and knowledge of number relationships, including place value, to determine relative size and position.	
	2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
Comparing Number: Mental Number Line to 5	Uses knowledge of counting number relationships to determine relative size and position when given perceptual support.	
Comparing Number: Object Corresponder	Puts objects into one-to-one correspondence, although may not understand that this creates equal groups (age 2; 8).	
Comparing Number: One-to-One Object Corresponder	Puts objects into 1-to-1 correspondence when it is clear the materials are a physical "pair." Implicitly sensitive to the relation of "more than/less than" involving very small numbers (from 1 to 2 years of age). Uses words to include "more," "less," or "same."	

Trajectory Level	Standards	
Comparing Number: Ordinal Counter	Identifies and uses ordinal numbers from “first” to “tenth.”	
Comparing Number: Perceptual Comparer	Compares collections that are quite different in number (e.g., one is at least twice the other). Compares similar collections but only involving very small numbers. Compares collections using number words “one” and “two” (age 2; 8).	
Comparing Number: Place Value Comparer	Compares numbers with place value understandings.	
	2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
	1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
	2.NBT.4	Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.
	1.NBT.3	Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.
Comparing Number: Scanning with Intuitive Quantification Estimator	Scans a group of objects and relates the results to a mental number line to perform a useful numerosity estimation.	

Trajectory Level	Standards
Comparing Number: Serial Orderer to 5	Orders quantities (dots) or numerals up to 5. Similarly orders lengths marked into units.
Comparing Number: Serial Orderer to 6+	Orders quantities (dots) or numerals to 6 and beyond. Similarly orders lengths marked into units.
	K.CC.7 Compare two numbers between 1 and 10 presented as written numerals.
Comparing Number: Spatial Extent Estimator	Extends sets and number categories to include “small numbers,” which are usually subitized, not estimated; “middle-size numbers” (e.g., 10–20); and “large numbers.” The arrangement of the to-be-estimated set affects the difficulty.
Comparing Number: Spatial Extent Estimator—Small/Big	Estimates which set is more or less if the differences are clear (e.g., one is double the other). Names a "small number" (e.g. from 1-4) for sets that cover little space and a "big number" (10-20 or more) for sets that cover a lot of space. Children classify numbers "little"/"big" idiosyncratically, and this may change with the size of the to-be-estimated objects.

Trajectory Level	Standards	
	K.CC.6	Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.
	1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Trajectory Level	Standards
Composing 2D Shapes	
Composing 2D Shapes: Picture Maker	Puts several shapes together to make one part of a picture (e.g., two shapes for one arm). Uses trial and error, and does not anticipate creation of new geometric shape. Chooses shapes using "general shape" or side length. Fills "easy" "Pattern Block Puzzles" that suggest the placement of each shape (note that in the example on the right the child is trying to put a square in the puzzle where its right angles will not fit).
	K.G.6 Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"
Composing 2D Shapes: Piece Assembler	Makes pictures in which each shape represents a unique role (e.g., one shape for each body part) and shapes touch. Fills simple puzzles in which all shapes are outlined, often using trial and error.
Composing 2D Shapes: Separate Shapes Actor: Foundations	Infants and toddlers manipulate shapes as individuals, but usually do not combine them to compose a larger shape.
Composing 2D Shapes: Shape Composer	Composes shapes with anticipation ("I know what will fit!"). Chooses shapes using angles as well as side lengths. Rotation and flipping are used intentionally to select and place shapes. In the "Pattern Block Puzzles" below, all angles are correct, and patterning is evident.

Trajectory Level	Standards	
	1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.4
Composing 2D Shapes: Shape Composer-Units of Units	Builds and applies units of units (shapes made from other shapes). For example, in constructing spatial patterns, extends patterning activity to create a tiling with a new unit shape—a unit of unit shapes that they recognize and consciously construct.	
Composing 2D Shapes: Shape Composite Repeater	Constructs and duplicates units of units (shapes made from other shapes) intentionally; understands each as being both multiple small shapes and one larger shape. May continue a pattern of shapes that leads to tiling.	
Composing 2D Shapes: Shape Decomposer (with Help)	Decomposes shapes using imagery that is suggested and supported by the task or environment.	
	1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.4
Composing 2D Shapes: Shape Decomposer with Imagery	Decomposes shapes flexibly using independently generated imagery. That is, decomposition is intentionally specified by the child.	

Trajectory Level	Standards	
Composing 2D Shapes: Shape Decomposer with Units of Units	Decomposes shapes flexibly using independently generated imagery and planned decompositions of shapes that themselves are decompositions.	
Composing 2D Shapes: Simple Decomposer	Decomposes ("takes apart" into smaller shapes) simple shapes that have obvious clues as to their decomposition.	
	K.G.6	Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"
Composing 2D Shapes: Substitution Composer	Makes new shapes out of smaller shapes, and uses trial and error to substitute groups of shapes for other shapes to create new shapes in different ways.	
	1.G.2	Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.4
	K.G.6	Compose simple shapes to form larger shapes. For example, "Can you join these two triangles with full sides touching to make a rectangle?"

Trajectory Level	Standards
Composing 3D shapes	
Composing 3D shapes: Line Maker	Shows use of relationship of "next to" to make a (one-dimensional) line of blocks.
Composing 3D shapes: Picture Maker (3D)	Uses multiple spatial relations, extending in multiple directions and with multiple points of contact among components, showing flexibility in integrating parts of the structure. Produces arches, enclosures, corners, and crosses, although may use unsystematic trial and error and simple addition of pieces.
Composing 3D shapes: Piece Assembler (3D)	Builds vertical and horizontal components within a building, but within a limited range, such as building a "floor" or a simple "wall." These, then, are two-dimensional structures.
Composing 3D shapes: Same Shape Stacker	Shows use of relationship of "on" to stack congruent blocks, or those that show a similarly helpful relationship to make stacks or lines.
Composing 3D shapes: Separate Blocks Actor: Foundations	Either places blocks randomly or manipulates shapes as individuals, but does not combine them to compose a larger shape. May pound, clap together, or use slide blocks or single blocks to represent an object, such as a house or truck.

Trajectory Level	Standards
Composing 3D shapes: Shape Composer - Unit of Units (3D)	Makes complex towers or other structures, involving multiple levels with ceilings (fitting the ceilings), and adult-like structures with blocks, including arches and other substructures.
Composing 3D shapes: Shape Composer (3D)	Composes shapes with anticipation, understanding what 3D shape will be produced with a composition of 2 or ore other (simple, familiar) 3D shapes. Can produce arches (with vertical interior space), enclosures (with internal horizontal space), corners, and crosses systematically. Builds enclosures and arches several blocks high. Later in this level, children add depth to make 3D structures, and they add roofs across structures multiple blocks high (but they may have no internal spaces).
Composing 3D shapes: Stacker	Shows use of the spatial relationship of “on” to stack blocks, although choice of blocks may be unsystematic.
Composing 3D shapes: Substitution Composer and Shape Composite Repeater (3D)	Substitutes a composite for a congruent whole. Builds complex bridges with multiple arches, ramps and stairs at the ends. Structures are 3D, often including roofs and multiple internal spaces.

Trajectory Level	Standards

Trajectory Level	Standards	
Composing Numbers		
Composing Numbers: Actor on Parts: Foundations	Displays actions that show intuition about parts and wholes such as gathering objects together. Only nonverbally recognizes parts and wholes. Recognizes that sets can be combined in different orders but may not explicitly recognize that groups are additively composed of smaller groups.	
Composing Numbers: Composer to 10	Knows number combinations to totals of 10. Quickly names parts of any whole, or the whole given parts. Doubles to 20.	
	K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
Composing Numbers: Composer to 4, then 5	Knows number combinations. Quickly names parts of any whole, or the whole given the parts.	
	K.OA.5	Fluently add and subtract within 5.
Composing Numbers: Composer to 7	Knows number combinations to totals of 7. Quickly names parts of any whole, or the whole given parts. Doubles to 10.	
Composing Numbers: Composer with Tens and Ones	Understands 2-digit numbers as tens and ones; count with dimes and pennies; 2-digit addition with regrouping.	

Trajectory Level	Standards	
	K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
	1.NBT.2.b	The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
Composing Numbers: Deriver +/-	Uses flexible strategies and derived combinations (e.g., “7 + 7 is 14, so 7 + 8 is 15) to solve all types of problems. Includes Break Apart to Make Ten (BAMT). Can simultaneously think of 3 numbers within a sum, and can move part of a number to another, aware of the increase in one and the decrease in another. Solves simple cases of multidigit addition (and, often, subtraction) by incrementing tens and/or ones.	
	K.OA.4	For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
Composing Numbers: Inexact Part-Whole Recognizer	Knows that a whole is bigger than parts, but may not accurately quantify (label with numbers). (May show intuitive knowledge of commutativity, and, later, associativity with physical groups, later in more abstract contexts, including numbers.)	
Composing Numbers: Multidigit +/-	Uses composition of tens and all previous strategies to solve multidigit +/− problems.	

Trajectory Level	Standards
Composing Numbers: Parts Combiner	Recognizes that sets can be combined in different orders, but may not explicitly recognize that groups are additively composed of smaller groups. The toddler also recognizes part-whole relations in nonverbal, intuitive, perceptual situations and can nonverbally represent parts that make a whole.
Composing Numbers: Problem Solver +/-	Solves all types of problems, with flexible strategies and known combinations. Multidigit may be solved by incrementing or combining tens and ones (latter not used for join, change unknown).

Trajectory Level	Standards
Counting	
Counting: Chanter	Chants number words in "sing-song" fashion and may run them together. The number words may be indistinguishable from one another ('onetwothree'). May begin a nonverbal object "counting" such as copying an adult's item-by-item placement of objects.
Counting: Corresponder	Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.
	K.CC.4.a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
Counting: Counter (10)	Counts arrangements of objects to 10 with understanding of the cardinal principle. May be able to read and write numerals to represent 1–10. May be able to tell the number just after or just before another number, but only by counting up from 1. Verbal counting to 20 is developing.
	PK.CC.1 Count verbally to 10 by ones. Count verbally to 3, then 5, by ones
	K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
Counting: Counter (Small Numbers)	Accurately counts objects in a line to 5 and answers the "how many" question with the last number counted, understanding that this represents the total number of objects (the cardinal principle).

Trajectory Level	Standards
	K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.
	K.CC.4.b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
Counting: Counter and Producer (10+)	Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements. Writes or draws to represent 1 to 10 (then, 20, then 30). Gives next number (usually to 20s or 30s). Separates the decade and the ones part of a number word and begins to relate each part of a number word/numeral to the quantity to which it refers. Recognizes errors in others' counting and can eliminate most errors in own counting (point-object) if asked to try hard.
	K.CC.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).
	K.CC.5 Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1-20, count out that many objects.
Counting: Counter Backward from 10	Counts backward from 10 to 1, verbally, or when removing objects from a group. "10, 9, 8, 7, 6, 5, 4, 3, 2, 1!"

Trajectory Level	Standards	
	K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality.
Counting: Counter Beyond 100	Accurately counts beyond 100, recognizing the patterns of ones, tens, and hundreds.	
	2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.
	1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
	2.NBT.1.b	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
	1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Counting: Counter Forward and Back	Counts “counting words” (single sequence or skip counts) in either direction. Recognizes that decades sequence mirrors single digit sequence. Switches between sequence and composition views of multidigit numbers easily.	
	2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.
Counting: Counter from N (N + 1, N -1)	Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the number of counts). Immediately determines numbers just after or just before.	
	K.CC.4.c	Understand that each successive number name refers to a quantity that is one larger.

Trajectory Level	Standards	
	1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
	K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
Counting: Counter of Quantitative Units/Place Value	Understands the base ten numeration system and place value concepts, including ideas of counting in units and multiples of hundreds, tens, and ones. When counting groups of ten, can decompose into 10 ones if that is useful. Understands value of a digit according to the place of the digit within a number. Counts unusual units, such as "wholes" when shown combinations of wholes and parts.	
	2.NBT.3	Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.
	K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
	2.NBT.1.b	The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).
	K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Trajectory Level	Standards
	2.NBT.1.a 100 can be thought of as a bundle of ten tens — called a “hundred.” **K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
	1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <. **K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
	1.NBT.2.c The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones). **K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
	1.NBT.2.b The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones. **K.NBT.1** Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Trajectory Level	Standards	
	1.NBT.2.a	10 can be thought of as a bundle of ten ones — called a "ten."
	K.NBT.1	Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.
Counting: Counter on Keeping Track	Counts forward or back from a given number keeping track of counting acts numerically, first using objects, then by "counting counts."	
	1.OA.5	Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).
	K.CC.4.c	Understand that each successive number name refers to a quantity that is one larger.
Counting: Counter on Using Patterns	Keeps track of counting acts, but only by using numerical patterns (spatial, auditory, or rhythmic) for adding 1 to about 3.	
	K.CC.2	Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
Counting: Counter to 100	Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number. "… 78, 79 … 80, 81 …"	
	K.CC.1	Count to 100 by ones and by tens.
	1.NBT.1	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.
Counting: Number Conserver	Consistently conserves number (i.e., believes number has been unchanged), even in face of perceptual distractions such as the spreading out of objects in a collection.	

Trajectory Level	Standards	
Counting: Number Word Sayer: Foundations	No verbal counting. Names some number words with no sequence.	
Counting: Producer (Small Numbers)	Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed. Produces a group of 4 objects.	
	K.CC.4	Understand the relationship between numbers and quantities; connect counting to cardinality.
Counting: Reciter	Verbally counts with distinct words, not necessarily in the correct order above "five." May say "One, two, three, four, five, seven." If knows more number words than number of objects, rattles them off quickly at the end; if more objects, "recycles" number words (inflexible list exhaustion)	
Counting: Reciter (10)	Verbally counts to ten with some correspondence with objects, but may either continue an overly rigid correspondence or exhibit performance errors (e.g., skipping, double-counting).	
Counting: Skip Counter	Counts by fives and twos with understanding.	

Trajectory Level	Standards	
	2.NBT.2	Count within 1000; skip-count by 5s, 10s, and 100s.
Counting: Skip Counter by 10s to 100	Skip counts by tens up to 100 or beyond with understanding; e.g., "sees" groups of 10 within a quantity and counts those groups by 10 (this relates to multiplication and algebraic thinking). "10, 20, 30 … 100."	
	K.CC.1	Count to 100 by ones and by tens.
	1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
	K.CC.1	Count to 100 by ones and by tens.
	1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
	1.NBT.2.c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
	1.NBT.5	Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
	1.NBT.2.c	The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).
	1.NBT.6	Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Trajectory Level	Standards

Trajectory Level	Standards
Disembedding Shapes	
Disembedding shapes: Complete Disembedder	Successfully identifies all varieties of complex arrangements.
Disembedding shapes: Intuitive Disembedder: Foundations	Can remember and reproduce only one or a small collection of non-overlapping (isolated) shapes.
Disembedding shapes: Secondary Structure Disembedder	Identifies embedded figures even when they do not coincide with any primary structures of the complex figure.
Disembedding shapes: Shapes in Shapes Disembedder	Identifies shapes embedded within other shapes, such as concentric circles and/or a circle in a square. Identifies primary structures in complex figures.
Disembedding shapes: Simple Disembedder	Identifies frame of complex figure. Finds some shapes in arrangements in which figures overlap, but not in those in which figures are embedded within others.

Trajectory Level	Standards	
Length		
Length: Conceptual Ruler Measurer	Possesses an “internal” measurement tool. Mentally moves along an object, segmenting it and counting the segments. Operates arithmetically on measures (“connected lengths”). Subdivides a unit at least into halves. Estimates with accuracy.	
	2.MD.3	Estimate lengths using units of inches, feet, centimeters, and meters.
	2.MD.6	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.
	3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
Length: End-to-End Length Measurer	Lays units end to end. May not recognize the need for equal-length units or be able to measure if there are fewer units that needed. The ability to apply resulting measures to comparison situations develops later in this level. (This develops in parallel with “Serial Orderer to 5 (Length)”).	
	1.MD.2	Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

Learning Trajectories: Alignment to CCSS

Trajectory Level	Standards	
Length: Length Direct Comparer	Physically aligns two objects to determine which is longer or if they are the same length. Uses terms: long, longer, longest.	
	K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
Length: Length Indirect Comparer	Compares the length of two objects by representing them with a third object. Uses terms: long, longer, longest, short, shorter, shortest. When asked to measure, may assign a length by guessing or moving along a length while counting (without equal-length units). May be able to measure with a ruler, but often lacks understanding or skill (e.g., ignores starting point).	
	1.MD.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.
	K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
Length: Length Measurer	Considers the length of a bent path as the sum of its parts (not the distance between the endpoints). Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers, and accumulation of distance. Begins to estimate.	
	2.MD.4	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.
	3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Trajectory Level	Standards	
	2.MD.1	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
	3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
	2.MD.5	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
	3.MD.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.
Length: Length Quantity Recognizer	Identifies length/distance as an attribute. May understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative (e.g., one person is taller than another). May compare non-corresponding parts of shape in determining side length.	
	K.MD.1	Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
Length: Length Senser: Foundations	Makes simple comparisons of length intuitively (similar to what we saw in Subitizing) as young as six months of age. However, may not recognize length as a distinct a	

Trajectory Level	Standards	
Length: Length Unit Relater and Repeater	Measures by repeating (iterating) a single unit and understands the need for equal-length unit. Relates the size and number of units (inverse relationship). Can add two lengths to obtain the length of a whole. Often can use rulers with minimal guidance in straightforward situations.	
	2.MD.2	Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
Length: Serial Orderer to 5	Orders lengths, marked in 1 to 5 units. Also, can compare unmarked lengths that are clearly different using broad categories ("big" and "small") and so can order 3 to 5 such objects but only by trial-and-error. With an increase in working memory, begins to build a mental image of the final ordering in which the lengths increase "bit by bit" with each successive length the smallest increase. This leads to more accurate and somewhat more efficient ordering. (This level develops in parallel with "End-to-End Length Measurer".)	
	K.MD.2	Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. For example, directly compare the heights of two children and describe one child as taller/shorter.
Length: Serial Orderer to 6+	Orders lengths, marked in 1 to 6 units. Understands at least intuitively that any set of objects of different lengths can be placed into a series that always increases (or decreases) in length, so spontaneously seriates with few errors by selecting the shortest (or longest) object, then the next shortest (the one with the "smallest difference"), and so forth.	
	1.MD.1	Order three objects by length; compare the lengths of two objects indirectly by using a third object.

Trajectory Level	Standards	
Multiplying / Dividing		
Multiplying/Dividing: Beginning Grouper and Distributive Sharer	Makes small groups (fewer than 5). Shares by “dealing out,” but usually only between 2 people. May not appreciate the numerical result.	
Multiplying/Dividing: Concrete Modeler x/÷	Solves small-number multiplying problems by grouping – making each group and counting all. Solves division/sharing problems with informal strategies, using concrete objects; up to 20 objects and 2-5 people. May not understand equivalence of groups.	
Multiplying/Dividing: Deriver X/÷	Uses strategies, patterns, de/composition (12 X 2 = 10 X 2 + 2 + 2) and derived combinations, such as multiplying X 9 as 10 – 1 or 7 X 8 from 7 X 7 + 7. Solves multidigit problems by operating on tens and ones separately.	
	3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.1
	3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.
Multiplying/Dividing: Grouper and Distributive Sharer	Makes small equal groups (fewer than 6). Deals out equally between two or more recipients, but may not understand that equal quantities are produced.	

Trajectory Level	Standards	
Multiplying/Dividing: Multidigit X/÷	(GONE) Uses multiple strategies, from compensating to paper-and-pencil procedures.	
	3.OA.5	Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
	3.OA.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8
Multiplying/Dividing: Nonquantitative Sharer: Foundations	Gives some, but not necessarily an equal number to each person.	
Multiplying/Dividing: Partitive Divisor	Figures out how many are in each group. May first repeatedly add a divisor until the dividend is reached.	

Trajectory Level	Standards	
	3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
Multiplying/Dividing: Parts and Whole X/÷	Understands the inverse relation between divisor and quotient in simple, concrete situations.	
	3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
	3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
	3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
	3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = � ÷ 3, 6 × 6 = ?
Multiplying/Dividing: Problem Solver X/÷	Solves many types of multiplicative problems, with flexible strategies and known combinations. Multidigit may be solved using combinations separately on ones and tens.	

Trajectory Level	Standards	
	3.OA.5	Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)
	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3
	3.OA.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8
	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3
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	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3
Multiplying/Dividing: Skip Counter X/÷	Uses repeated adding, additive doubling, or skip counting to solve multiplication and for measurement division (finding out how many groups) problems. Uses trial and error for partitive division (finding out how many in each group).	

Trajectory Level	Standards	
	2.OA.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
	3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.3
	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.
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	3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Trajectory Level	Standards
Patterning	
Patterning: Beginning Arithmetic Patterner	Recognizes and uses arithmetic patterns with perceptual or pedagogical support, often first those that involve properties of zero. The child also accepts number sentences not in the form of 3 + 4 = 7 (e.g., 7 = 3 + 4, or even 3 + 4 = 2 + 5). This represents a move from an “equals-as-an answer” notion to recognizing that equals means equivalent numbers. In functional thinking, builds two sets (e.g., in a t-chart) following two separate general rules, level 3.
Patterning: Intuitive Patterner: Foundations	Detects and uses patterning implicitly and intuitively, such as in movement activities or common nursery rhymes that repeat words and action. May be attentive to repeating patterns without recognizing them explicitly or accurately, often attending to individual attributes such as color.
Patterning: Numeric Patterner	Describes a pattern numerically, can translate between geometric and numeric representation of a series. In functional thinking, builds and perceives a t-chart as a sequence of particular instances, level 2.
Patterning: Pattern Recognizer	Recognizes a simple pattern, usually ABABAB, as a pattern, even if doesn’t yet name or describe it.

Trajectory Level	Standards
Patterning: Pattern Translator and Unit Recognizer	Translates patterns into new media or using new materials; that is, abstract and generalize the pattern. Identifies the smallest core unit of a repeating pattern. (Most research indicates this develops later, Miller et al., 2016.) In functional thinking situations (e.g., p. #), does not yet see math relationships in sets of data, level 1.
Patterning: Patterner	Recognizes, describes, and builds repeating patterns, including AB but also patterns with core units such as AAB, ABC, and AABC.
Patterning: Patterner AB	Recognizes, describes, and builds repeating ABAB patterns. These involve the following, which many children learn in this order, although this can vary by the task.1 Fixes AB: Fills in missing element of an ABAB pattern. Duplicates AB: Duplicates ABABAB pattern (at first may have to work close to the model pattern, but eventually can build the same pattern away from the model pattern or when the model is out of sight). Extends AB: Extends AB patterns to add multiple units to the end of the pattern. This is easier for children if the pattern ends with a complete unit, but they eventually learn to extend those that end with a partial unit.
Patterning: Relational Thinker +/-	Recognizes and uses patterns that involve addition and subtraction and, understanding equality, can compare two sides of a number sentence with reasoning, and thus does not have to carry out computations. In functional thinking, creates functional relationships between two data sets but only for specific cases (Blanton et al., 2015)level 4. May use letters to represent numbers, but only as representing objects or fixed values.

Trajectory Level	Standards
Patterning: Relational Thinker with Multiplication	Recognizes and uses patterns that involve multiplication as repeated addition and use of the distributive property to partition number facts. In functional thinking, generalizes functional relationships between two data sets, (Blanton et al., 2015). level 7. Uses letters as variables represent this relationhip≥
Patterning: Relational Thinker-Symbolic +/-	Recognizes and uses patterns that involve addition and subtraction and anunderstanding of equality. Can compare two sides of a number sentence with reasoning, even when the quantities are represented by variables, such as a + b = b + a. . In functional thinking, generalizes functional relationships between two data sets, at first just noticing, and later a quantitative relationship (Blanton et al., 2015). level 5, 6 Uses letters for unknown numbers, an initial algebraic notion.

Trajectory Level	Standards	
Shapes		
Shapes: "Same Thing" Comparer: Foundations	Compares real-world objects	
Shapes: Angle Recognizer—More Contexts	Recognizes and describe contexts in which angle knowledge is relevant, including corners (can discuss “sharper” angles), crossings (e.g., a pair of scissors), and, later, bent objects and bends (sometimes bends in paths and slopes). Only later can explicitly understand how angle concepts relate to these contexts (e.g., initially may not think of bends in roads as angles; may not be able to add horizontal or vertical to complete the angle in slope contexts; may even see corners as more or less “sharp” without representing the lines that constitute them). Often does not relate these contexts and may represent only some features of angles in each (e.g., oblique line for a ramp in a slope context).	
	1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
Shapes: Angle Representer	Represents various angle contexts as two lines, explicitly including the reference line (horizontal or vertical for slope; a “line of sight” for turn contexts) and, at least implicitly, the size of the angle as the rotation between these lines (may still maintain misconceptions about angle measure, such as relating angle size to the length of side’s distance between endpoints, and may not apply these understandings to multiple contexts).	
Shapes: Angle Synthesizer	Combines various meanings of angle (turn, corner, slant), including angle measure.	

Trajectory Level	Standards
Shapes: Congruence Representer	Refers to geometric properties and explains with transformations.
Shapes: Congruence Superposer	Moves and places objects on top of each other to determine congruence. Can also determine congruence by comparing all attributes and all spatial relationships.
	2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.5 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
Shapes: Constructor of Shapes From Parts Exact	Uses manipulatives representing parts of shapes, such as sides and angle “connectors,” to make a shape that is completely correct, based on knowledge of components and properties — relationships between the components.
	2.G.1 Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.5 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
Shapes: Constructor of Shapes from Parts—Looks Like	Uses manipulatives representing parts of shapes, such as sides, to make a shape that “looks like” a goal shape. May think of angles as a corner (which is “pointy”).

Trajectory Level	Standards	
	K.G.5	Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
Shapes: Corner (Angle) Recognizer	Recognizes angles as separate geometric objects, at least in the limited context of “corners.”	
	K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
Shapes: Parts of Shapes Identifier	Identifies shapes in terms of their components.	
Shapes: Property Class Identifier	Uses class membership for shapes (e.g., to sort or consider shapes “similar”) explicitly based on properties, including angle measure. Is aware of restrictions of transformations and also of the definitions and can integrate the two. Sorts hierarchically, based on properties.	
	3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
Shapes: Shape Class Identifier	Uses class membership (e.g., to sort), not explicitly based on properties.	

Trajectory Level	Standards	
	3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
Shapes: Shape Identifier	Names most common shapes, including, for example, rhombuses, hexagons, octagons, and trapezoids, without making mistakes, such as calling ovals "circles." Recognizes (at least) right angles, so distinguishes between a rectangle and a parallelogram without right angles.	
	2.G.1	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.5 Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
	1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
Shapes: Shape Matcher-Identical, Orientations, Sizes	Identical - Matches familiar shapes (circle, square, typical triangle) with same size and orientation. Sizes - Matches familiar shapes with different sizes. Orientations - Matches familiar shapes with different orientations.	
Shapes: Shape Matcher—More Shapes, Sizes, Orientations, & Combinations	Matches a wider variety of shapes with same size and orientation. Matches a wider variety of shapes with different sizes and orientations. Matches combinations of shapes to each other.	
	K.G.2	Correctly name shapes regardless of their orientations or overall size.

Trajectory Level	Standards	
Shapes: Shape Property Identifier	Uses properties explicitly. Can see the invariants in the changes of state or shape, but maintaining the shapes' properties.	
	3.G.1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
Shapes: Shape Recognizer-All Rectangles	Recognizes rectangles of all sizes, shapes, and orientations.	
	K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
Shapes: Shape Recognizer—Circles, Squares, and Triangles	Recognizes some less typical squares and triangles and may recognize some rectangles, but usually not rhombuses (diamonds). Often doesn't differentiate sides/corners.	
Shapes: Shape Recognizer—More Shapes	Recognizes most familiar shapes and typical examples of other shapes, such as hexagon, rhombus (diamond), and trapezoid.	
	1.G.1	Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.

Trajectory Level	Standards	
Shapes: Shape Recognizer-Typical	Recognizes and names a typical circle, square, and, less often, triangle. May physically rotate shapes in atypical orientations to mentally match them to a prototype.	
Shapes: Side Recognizer	Identifies sides as distinct geometric objects with attributes.	
	K.G.3	Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").
	K.G.4	Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).

Trajectory Level	Standards
Spatial Orientation	
Spatial Orientation: Coordinate Plotter	Reads and plots coordinates on maps.
Spatial Orientation: Foundations of Spatial Orientation	Uses the earliest of two types of cognitive systems for spatial orientation—knowing where you are and how to get around in the world. 1. Response Learning: Uses the first self-based system – that is, related to the child's own position and movements. Notes a pattern of movements that have been associated with a goal. 2. Cue Learning: Uses the first external-based systems, based on familiar landmarks.
Spatial Orientation: Framework User	Uses general frameworks that include the observer and landmarks. May not use precise measurement even when that would be helpful, unless guided to do so. Can follow and create maps, even if spatial relations are transformed.
Spatial Orientation: Local Framework User	Locates objects after moving, maintaining the overall shape of the arrangement of objects. Represents objects' positions relative to landmarks (e.g., about halfway in between two landmarks) and keeps track of own location in open areas or mazes. Uses spatial vocabulary to direct attention to spatial relations. Uses coordinate labels in simple situations such as games.

Trajectory Level	Standards
	K.G.1 Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.
Spatial Orientation: Local-Self Framework User	Uses distant landmarks to find objects or location near them, even after they have moved themselves relative to the landmarks, if the target object is specified ahead of time. Orients a horizontal or vertical line in space (Rosser, Horan, Mattson, & Mazzeo, 1984). Uses spatial vocabulary to direct attention to spatial relations, including more difficult terms such as "beside" and "between."
Spatial Orientation: Map User	Locates objects using maps with pictorial cues. Extrapolates (extends) two coordinates, understanding the integration of them to one position, as well as use coordinate labels in simple situations.
Spatial Orientation: Path Integrater	Remembers and can repeat movements they have made including the approximate distances and directions.
Spatial Orientation: Place Learner	Creates “mental maps” by storing locations, distances, and directions to landmarks and solves spatial problems. Uses the walls of a room as a frame of reference; uses spatial vocabulary, such as "in," "on," and "under," along with vertical directionality terms as "up" and "down."

Trajectory Level	Standards	
Spatial Orientation: Route Map Follower	Follows a simple route map, with more accurate direction and distances.	
Spatial Orientation: Small Local Framework User	Locates objects after movement, even if target is not specified ahead of time. Searches a small area comprehensively, often using a circular search pattern. Uses words referring to frames of reference such as "in front of" and "behind" or "left" and "right." In meaningful graphing contexts, extrapolates lines from positions on both axes (like a coordinate grid) and determines where they intersect.	
	K.G.1	Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind, and next to.

Trajectory Level	Standards
Spatial Visualization	
Spatial Visualization: Beginning Slider, Flipper, Turner	Uses the correct motions guided by more developed intuition, but not always accurate in direction and amount (adjusts these with trial and error). Knows a shape has to be flipped to match another shape, but flips it in the wrong direction.
Spatial Visualization: Concrete Slider, Flipper, Turner	Can move shapes to a location by physical trial and error.
Spatial Visualization: Diagonal Mover	Performs diagonal slides and flips as well as all motions from previous levels. Knows a shape must be turned flipped over an oblique line (45° orientation) to fit into a puzzle.
Spatial Visualization: Intuituitive Mover: Foundations	Explores the size and shape of objects by observing them as they move in space, using trial and error to discover how they fit into space, and eventually predicting what will fit inside a space without attempting all possible solutions. Such skills will eventually support future spatial visualization.
Spatial Visualization: Mental Mover	Predicts results of moving shapes using mental images (any direction or amount).

Trajectory Level	Standards
Spatial Visualization: Simple Slider and Turner	Slides and turns objects accurately in easy tasks, guided by an early intuition that starts the motion and then adjusts (the motion, direction, or amount1) in real time as the motion is carried out.
Spatial Visualization: Slider, Flipper, Turner	Performs slides and flips, often only horizontal and vertical, using manipulatives but guided by mental images of these motions (of turns of 45, 90, and 180° and flips over vertical and horizontal lines). That is, they can mentally imagine the motion and the result of it. Knows a shape must be turned 90° to the right to fit into a puzzle.

Trajectory Level	Standards	
Subitizing		
Subitizing: Conceptual Subitizer to 10	erbally labels most briefly shown arrangements of all numbers 2 to 10. Children may know some familiar ones (“5 and 5 make 10” is common) early, but this level is reached when most all combinations of all numbers up to 10 are recognized (e.g., 7 and 2 seen as 9; 5 and 3 seen as 8; etc.). Uses structures such as tens-frames to recognize larger quantities.	
Subitizing: Conceptual Subitizer to 20	Verbally labels structured arrangements of 10 to 20, shown only briefly, by seeing the parts and quickly knowing the whole. Spontaneously makes use of a top-down strategy to subitizing large quantities (Nes, 2009). Children may know some familiar ones (“10 and 10 make 20” is common) early, but this level is reached when most all combinations of numbers from 1 to 10 are recognized (e.g., 7 and 9 is seen as 16).	
	1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	1.OA.6	Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).
	1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	1.OA.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = � – 3, 6 + 6 = �.

Trajectory Level	Standards	
	1.OA.2	Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	2.OA.2	Fluently add and subtract within 20 using mental strategies.2 By end of Grade 2, know from memory all sums of two one-digit numbers.
Subitizing: Conceptual Subitizer to 5	Verbally labels all arrangements to about 5, shown only briefly, by seeing the parts and quickly knowing the whole. Conceptual subitizing refers to the ability of children to identify a whole quantity as a result of composing smaller quantities (recognized through perceptual subitizing) that make up the whole.	
	K.OA.3	Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
	K.OA.5	Fluently add and subtract within 5.
Subitizing: Conceptual Subitizer to 7	Verbally labels all arrangements to 6, then 7, when shown only briefly.	
Subitizing: Conceptual Subitizer with Place Value	Verbally labels structured arrangements, shown only briefly, using groups, skip counting, and place value.	
	2.NBT.5	Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
	2.NBT.8	Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

Trajectory Level	Standards
	1.NBT.2.a 10 can be thought of as a bundle of ten ones — called a “ten.” **2.NBT.8** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
	2.NBT.1.a 100 can be thought of as a bundle of ten tens — called a “hundred.” **2.NBT.8** Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.
Subitizing: Conceptual Subitizer with Place Value and Multiplicative Thinking	Verbally labels structured arrangements, shown only briefly, using groups, multiplicative thinking, and place value. This level builds on the previous level, such that children are able to use the base-10 system to conceptually subitize larger numbers. Children are able to verbalize the quantity of 10's they see.
Subitizing: Maker of Small Collections	Makes a small collection (usually 1–2 and possibly 3) with the same number as another collection (via mental model; i.e., not necessarily by matching—for that process, see Compare Number). Might also be verbal but often is not. May not recognize spatial structures at first, and may count this.
Subitizing: Number Senser	Has inborn specific "sensers" for number from the first months of life without explicit knowledge of number. Intuitively distinguishes between groups of 1 and of 2 (and possibly 2 and 3). Also shows sensitivity to ratios of quite large numbers. (Approximate Number System, or ANS). These are pre-math, foundational abilities.

Learning Trajectories: Alignment to CCSS

Trajectory Level	Standards
Subitizing: Perceptual Subitizer to 4	Instantly recognizes collections up to 4 briefly shown and verbally names the number of items.
Subitizing: Perceptual Subitizer to 5	Instantly recognizes briefly shown collections up to 5 and verbally names the number of items. Recognizes and uses spatial and numeric structures beyond the situations in which they were already experienced (i.e., in which they were initially learned).
	K.OA.5 Fluently add and subtract within 5.
Subitizing: Small Collection Namer	Names groups of 1, 2, and 3 with increasing accuracy. Most children of about 34-39 months of age can accurately name groups of 1, 2, and 3. Many children learn to recognize and name groups of 4 about 6 months later. For a Maker of Small Collections (the previous level), the child may rely on matching strategies to make their small collection. In Small Collection Namer, the child is actually able to recognize small groups without relying on a model or matching strategy.
Subitizing: Very Small Number Recognizer	Begins connecting small quantities to number words to form an explicit idea of cardinality, or “how-many-ness.” Following the child's first birthday, the number words “one” and “two” are often learned. Other general terms such as “more” and "less" usually follow. Only over time do they begin to understand that all groups labelled with the same number word have the same amount.

Trajectory Level	Standards
Volume	
Volume: 3-D Array Structurer	Has an abstract understanding of the rectangular prism volume formula. Shows a propensity for multiplicative comparisons, coordinates multiplicative and additive comparisons flexibly. With linear measures or other similar indications of the three dimensions, multiplicatively iterates cubes in a row, column, and/or layers to determine the area. Constructions and drawings are not necessary. In multiple contexts, children can compute the volume of rectangular prisms from its dimensions and explain how that multiplication creates a measure of volume.
Volume: 3-D Row and Column Structurer	Able to coordinate flexibly filling, packing, building aspects of volume. Shows a propensity for additive comparisons (e.g., “this one has 12 more”) but may show some nascent multiplicative comparisons (e.g., “this one is four times as big”). Initially counts or computes (e.g., number of rows times number of columns) the number of cubes in one layer, and then uses addition or skip counting by layers to determine the total volume. Eventually moves to multiplication (e.g., number of cubes in a layer times number of layers).
Volume: Initial Composite 3-D Structurer	Understands cubes as filling a space but does not use layers or multiplicative thinking. Moves to more accurate counting strategies. Relates number of cubes to cubic units as measured by capacity. Given a graduated cylinder marked in cubic-inch units, child understands that sand filled to the 10 in the cylinder would fill a box that holds ten, 1-inch cubes. Begins to visualize and operate on composite units such as rows or columns (what we call a 1x1xn core). Iterates to pack the space completely, accounting for “internal/ hidden” cubes. Decomposes space, allowing for accurate use of units and subunits. Recognizes when a box is half full, visualizes remaining rows or columns.

Trajectory Level	Standards
Volume: Volume Filler	Can compare two containers by pouring one into the other (although can be confused at “which holds more” at first). Fills a container using another (smaller container) and counts the number needed to completely fill the larger container (but may not use accurately filled scoops and may not focus on quantifying the total volume or capacity). In packing situations, places cubes into a rectangular box to fill it. Eventually packs entire box with cubes in an organized way. Compares objects by physically or mentally aligning; refers to at least two dimensions of objects. May be able to compare two containers using a third container and transitive reasoning.
Volume: Volume Quantifier	Partial understanding of cubes as filling a space. Able to estimate number of scoops needed to fill. Able to attend to both the portion of container filled and the portion remaining unfilled. Recognizes when container is half full. Exhibits initial spatial structuring. Packs box neatly and completely with cubes; may count one cube at a time, while packing, to determine total. Compares objects by physically or mentally aligning and explicitly recognizing three dimensions.
Volume: Volume Quantity Recognizer	Identifies capacity or volume as attribute. Builds with blocks, associating more blocks with terms like “big” and fewer blocks with terms like “small.”

Learning Trajectories: Alignment to CCSS

Trajectory Level	Standards
Volume: Volume Senser: Foundations	Sensitive to volume even in the first year; however, they may not for some time explicitly recognize volume as an attribute (separate from general size, such as “small” and “big”).
Volume: Volume Unit Relater and Repeater	Uses simple units to fill containers, with accurate counting. Relates size and number of units explicitly; understands that fewer larger than smaller units will be needed to fill or pack a given container. Can accurately convert units in 1:2 ratio.