



Trajectory Level

Adding / Subtracting

Adding & Subtracting: Arithmetic Senses: Foundations

Very young children are sensitive to combining or separating perceptual groups. An infant may observe, point, or make sounds while someone else introduces a quantity of objects. They will notice the effects of increasing or decreasing small collections by one item and may also be sensitive to the results of combining larger groups.

TSG Objectives

Adding & Subtracting: Counting Strategies +/-

Finds sums for joining (“You had 8 apples and get 3 more ...”) and part–part–whole (“6 girls and 5 boys ...”) problems with finger patterns and/or by counting on.

TSG Objectives

20.e.2	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves addition and subtraction word problems of whole numbers within 10 using a variety of strategies (counting objects or fingers, counting on, counting back); makes number pairs within 10
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TSG Objectives

20.f.4	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts whole numbers fluently within 10 using mental strategies (counting on, making ten, decomposing/recomposing, addition/subtraction relationship, and easier equivalent known sums)
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Adding & Subtracting: Deriver +/-

Uses flexible strategies and derived combinations (e.g., “7 + 7 is 14, so 7 + 8 is 15”) to solve all types of problems. Includes “Break Apart to Make Ten” (BAMT; explained in Chapter 6). Can simultaneously think of 3 numbers within a sum, and can move part of a number to another, aware of the increase in one and the decrease in another. May solve simple cases of multidigit addition (sometimes subtraction) by counting by tens and/or ones.

TSG Objectives

20.f.6	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts whole numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory
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Trajectory Level

TSG Objectives

20.d.4	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write two-digit numbers, add one- and two- digit numbers (within 100), and subtract multiples of 100 from multiples of 10 (10 – 90)
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TSG Objectives

20.d.3	Uses number concepts and operations	Understands and uses place value and base ten
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TSG Objectives

20.e.5	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.e.4	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves three-number word problems with answers within 20 using addition properties (associative, communicative, and additive); solves addition and subtraction equations of different types with unknowns in various positions for amounts up to 20
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Adding & Subtracting: Find Change +/-

Finds the missing addend (e.g., $5 + _ = 7$ or $9 - _ = 3$) to solve Join and Separate, Change Unknown problems by adding on or taking away objects. Compares by matching in simple situations.

TSG Objectives

20.e.2	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves addition and subtraction word problems of whole numbers within 10 using a variety of strategies (counting objects or fingers, counting on, counting back); makes number pairs within 10
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TSG Objectives

20.f.4	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts whole numbers fluently within 10 using mental strategies (counting on, making ten, decomposing/recomposing, addition/subtraction relationship, and easier equivalent known sums)
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Trajectory Level

Adding & Subtracting: Find Result +/-

Finds sums for Join, Result Unknown problems (“You had 3 apples and get 3 more, how many do you have in all?”) and part–part–whole (“There are 6 girls and 5 boys on the playground, how many children were there in all?”) problems by direct modeling, counting all, with objects. Solves take-away problems by separating with objects.

TSG Objectives

20.b.7 Uses number concepts and operations Quantifies

TSG Objectives

20.b.8 Uses number concepts and operations Quantifies Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts

TSG Objectives

20.f.4 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts while numbers fluently within 10 using mental strategies (counting on, making ten, decomposing/recomposing, addition/subtraction relationship, and easier equivalent known sums)

TSG Objectives

20.b.9 Uses number concepts and operations Quantifies

TSG Objectives

20.b.6 Uses number concepts and operations Quantifies Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many

TSG Objectives

20.e.2 Uses number concepts and operations Applies properties of mathematical operations and relationships Solves addition and subtraction word problems of whole numbers within 10 using a variety of strategies (counting objects or fingers, counting on, counting back); makes number pairs within 10

Trajectory Level

Adding & Subtracting: Make it N

Adds on objects to “make one number into another,” without needing to count from 1. Does not (necessarily) represent how many were added (this is not a requirement of this intermediate-difficulty problem type)

TSG Objectives

20.e.2	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves addition and subtraction word problems of whole numbers within 10 using a variety of strategies (counting objects or fingers, counting on, counting back); makes number pairs within 10
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TSG Objectives

20.f.4	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 10 using mental strategies (counting on, making ten, decomposing/recomposing, addition/subtraction relationship, and easier equivalent known sums)
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TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental)
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TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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TSG Objectives

20.d.8	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write four-digit numbers; multiplies one-digit whole numbers by 10s (10 – 90); rounds three-digit whole numbers to the nearest ten or hundred
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Trajectory Level

TSG Objectives

20.d.9 Uses number concepts and operations Understands and uses place value and base ten

TSG Objectives

20.e.6 Uses number concepts and operations Applies properties of mathematical operations and relationships Solves one- and two- step word problems of various types using addition and subtraction (within 100) and explains strategies; uses repeated addition to find the number of objects presented in rectangular arrays (up to five rows and five columns)

TSG Objectives

20.e.9 Uses number concepts and operations Applies properties of mathematical operations and relationships

TSG Objectives

20.f.9 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations

TSG Objectives

20.d.7 Uses number concepts and operations Understands and uses place value and base ten

TSG Objectives

20.d.5 Uses number concepts and operations Understands and uses place value and base ten

TSG Objectives

20.d.6 Uses number concepts and operations Understands and uses place value and base ten Uses place-value understanding to represent and write three-digit numbers (including expanded form); adds up to four two-digit numbers; adds and subtracts three-digit numbers (within 1,000)

Trajectory Level

TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental)
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TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.f.8	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on
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Adding & Subtracting: Numbers-in-Numbers +/- Recognizes when a number is part of a whole and can keep the part and whole in mind simultaneously; solves “Start Unknown (e.g., $_ + 4 = 9$) problems with counting strategies.

TSG Objectives

20.d.2	Uses number concepts and operations	Understands and uses place value and base ten	Indicates base-ten equivalents for numbers 11 – 19 using objects and drawings; may use simple equations
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TSG Objectives

20.f.5	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations
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TSG Objectives

20.e.4	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves three-number word problems with answers within 20 using addition properties (associative, communicative, and additive); solves addition and subtraction equations of different types with unknowns in various positions for amounts up to 20
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Trajectory Level

TSG Objectives

20.f.6	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory
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Adding & Subtracting: Part-Whole +/-

Has initial part-whole understanding and can solve all previous problem types using flexible strategies. May use some known combinations, such as $5 + 5$ is 10. Sometimes can do “Start Unknown (e.g., $_ + 6 = 11$), but only by trial and error.

TSG Objectives

20.f.5	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations
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TSG Objectives

20.f.6	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory
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TSG Objectives

20.e.3	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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Adding & Subtracting: Preverbal +/-

Adds and subtracts very small collections (totals up to three), often making a collection rather than answering verbally.

TSG Objectives

Adding & Subtracting: Problem Solver +/-

Solves all types of problems, with flexible strategies and known combinations. Multidigit may be solved by incrementing tens and ones by counting (latter not used for Join, Change Unknown).

TSG Objectives

Adding & Subtracting: Small Number +/-

Finds sums for Join, Result Unknown and Separate, Result Unknown problems with totals up to 5 by “counting all” with objects.

Trajectory Level

TSG Objectives

20.b.6	Uses number concepts and operations	Quantifies	Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many
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TSG Objectives

20.e.1	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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TSG Objectives

20.b.5	Uses number concepts and operations	Quantifies	
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TSG Objectives

20.b.4	Uses number concepts and operations	Quantifies	Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts
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Trajectory Level

Angle and Turn

Angle and Turn measurement:
Angle and Turn Senser:
Foundations

Infants are sensitive to angles-as-turning, both turning objects and their own body. See more at the first three levels of the Spatial Orientation and the first level of Spatial Visualization.

TSG Objectives

Angle and Turn measurement:
Angle Matcher

Matches angles concretely. Explicitly recognizes parallels from non-parallels in specific contexts (Mitchelmore, 1992). Sorts angles into “smaller” or “larger” (but may be misled by irrelevant features, such as length of line segments).

TSG Objectives

Angle and Turn measurement:
Angle Measurer

Understands angle and angle measure in both primary aspects, and can represent multiple contexts in terms of the standard, generalizable concepts and procedures of angle and angle measure (e.g., two rays, the common endpoint, rotation of one ray to the other around that endpoint, and measure of that rotation).

TSG Objectives

Angle and Turn measurement:
Angle Size Comparer

Differentiates angle and angle size from shapes and contexts, and compares angle sizes. Recognizes right angles, and then equal angles of other measures, in different orientations. Compares simple turns. (Note that, without instruction, this and higher levels may not be achieved even by the end of the elementary grades.)

TSG Objectives

Angle and Turn measurement:
Implicit Angle User

Uses angles and, at least implicitly, some angle measure concepts, such as parallelism and perpendicularity—in physical alignment tasks, construction with blocks, or other everyday contexts. May identify corresponding angles of a pair of congruent triangles using physical models. Uses the word “angle” or other descriptive vocabulary to describe some of these situations.

Trajectory Level

TSG Objectives

Angle and Turn measurement: Intuitive Angle Builder

Intuitively uses some angle measure notions in everyday settings, such as building with blocks, solving puzzles, and walking.

TSG Objectives

Trajectory Level

Area

Area: Area Quantity Recognizer

Perceives the amount of two-dimensional space and can make intuitive comparisons. However, when asked to compare, may compare lengths more than areas because lengths are salient and familiar to them (e.g., compare one side of one piece of paper to the side of another) or make estimates based on a “length plus (not times) width” intuition. However, may compare areas correctly if the task suggestions superposition (putting one on top of the other). Asked to partition a space into squares or copy an image of a rectangle partitioned into an array (rows and columns), may simply draw squares (usually!) inside the rectangle or other types of shapes or short paths on or around the rectangle.

TSG Objectives

22.a.4	Compares and measures	Measures objects	Compares and orders a small set of objects as appropriate according to size, length, weight, area, or volume.
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TSG Objectives

22.a.3	Compares and measures	Measures objects
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Area: Area Row and Column Structurer

Decomposes and recomposes partial units to make whole units. For example, draws rows as rows making parallel horizontal lines and so forth. Begins conserving area and reasons about additive composition of areas (e.g., how regions that look different can have the same area measure) and recognizes the need for space-filling in most contexts.

TSG Objectives

22.b.1	Compares and measures	Measures Time and Money
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TSG Objectives

22.b.2	Compares and measures	Measures Time and Money	Knows usual sequences of basic daily events
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TSG Objectives

22.a.14	Compares and measures	Measures objects	Solves one-step word problems related to measurement of liquid volume, mass, area, and perimeter.
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Trajectory Level

TSG Objectives

22.a.15 Compares and Measures objects
measures

TSG Objectives

22.a.13 Compares and Measures objects
measures

Area: Area Senser: Foundations

Even children in their first year are sensitive to area. However, may not explicitly recognize area as an attribute (separate from general size, such as “small” and “big”) for some time. If asked to fill in a rectangle, preschoolers may just draw approximations of circles (Mulligan, Prescott, Mitchelmore, & Outhred, 2005). Uses side matching strategies in comparing areas.

TSG Objectives

Area: Area Unit Relater and Repeater

Counts individual units, often trying to use the structure of rows. To cover a region with physical units, repeats (iterates) an individual unit. Draws a complete covering based on an intuitive notion of rows and columns, making equal sized units, but often draws them one at a time. That is, draws individual, mainly equal-sized units that are lined up but may not see groups of units making up individual rows or columns. Relates the size and number of units to cover a region, recognizing that differently sized units will result in different measures and that the larger the unit, the fewer will be needed. Compares areas by accurately counting units in each and comparing the resulting measures.

TSG Objectives

22.a.10 Compares and Measures objects
measures

Measures length accurately and expresses the measurement in whole numbers

TSG Objectives

22.a.11 Compares and Measures objects
measures

TSG Objectives

22.a.9 Compares and Measures objects
measures

Trajectory Level

Area: Array Structurer

With linear measures or other similar indications of the two dimensions, multiplicatively iterates rows or columns to determine the area. Does not need to draw in the array to do so. Has an abstract understanding of the rectangular area formula. Understands and justifies that differently-shaped regions can have the same areas. Compares regions with transitive reasoning (e.g., A is greater than B, B is greater than C, so I know A is greater than C).

TSG Objectives

Area: Complete Coverer and Counter

Draws a complete covering of a specific region without gaps or overlaps and in approximations of rows. When provided with more than the total number of physical tiles needed, can build a region of specified area (e.g., build a rectangle with an area of 12 from a pile of 20 tiles).

TSG Objectives

Area: Initial Composite Structurer

Identifies a square unit as both a unit and a component of a larger unit of units (a row, column, or group) and uses those structures in counting or drawing. However, needs figural support to structure the space themselves (this may include physical motions of some of the tiles or drawing some collections of units rather than from using the dimensions). At this level, usually does not coordinate the width and height and in measuring, may not use the dimensions of the rectangle to constrain the unit size. Makes reasonable estimates of areas.

TSG Objectives

Area: Physical Coverer and Counter

Prompted to measure, attempts to cover a rectangular space with physical tiles. However, doesn't organize or structure the 2D space without considerable perceptual support, such as a grid that outlines each individual unit. In drawing (or imagining and pointing to count squares as units of area), represents only certain aspects of that structure, such as approximately rectangular shapes next to one another. Makes comparison areas based on simple, direct comparisons (e.g., a child places one sheet of paper over another piece of paper to select the sheet that covers more space).

Trajectory Level

TSG Objectives

22.a.4

Compares and Measures objects
measures

Compares and orders a small set of objects as appropriate according to size, length, weight, area, or volume.

Trajectory Level

Classification and Data Analysis

Classification: Consistent, Flexible Sorter

Sorts consistently by a single attribute and re-classifies by different attributes. Sorts consistently and exhaustively by an attribute, given or created, and uses the terms “some” and “all.”

TSG Objectives

22.c.2	Compares and measures	Represents and analyzes data	Knows a few ordinal numbers
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TSG Objectives

22.c.3	Compares and measures	Represents and analyzes data
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Classification: Data Aggregator

Classifies objects that may be perceptually different by more abstract attributes such as function or conceptual attributes. Focuses on features of the data set as a whole. Uses to describe relative frequency and density (shape), and location (centers). Begins to understand the concepts of expectation (averages and probabilities) and variation (“spread” of values). Views ranges in data or view the mode (the number or range of numbers that occurs most frequently). Eventually, can focus on features of the data set as a whole, including the relative frequencies, density (“shape”), and location (centers, such as the mean).

TSG Objectives

22.c.7	Compares and measures	Represents and analyzes data
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TSG Objectives

22.c.6	Compares and measures	Represents and analyzes data	Organizes, represents, and analyzes data with up to three categories; uses simple numerical summaries (counts, tallies) and ordinal terms to describe findings
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Classification: Data Case Viewer

Associates a value with an individual case. Uses numeric data to identify largest/smallest cases. Before this level, children may be “pointers” in which data records point to the entire event (“We talked about favorite colors”). They use it like string tied around a finger, to remember that they did something.

Trajectory Level

TSG Objectives

22.c.3 Compares and measures Represents and analyzes data

TSG Objectives

22.c.2 Compares and measures Represents and analyzes data Knows a few ordinal numbers

Classification: Data Classifier

Data Treats cases with similar values as the same. Uses to compare category frequencies (most and least popular case-types). Visually compares two graphs.

TSG Objectives

22.c.4 Compares and measures Represents and analyzes data Creates and reads simple graph; uses simple comparison and ordinal terms to describe findings

Classification: Data Representer

Shows an appreciation of the “center” of graphs and for their variation or spread. Compares graphs of data sets of the same size accurately.

TSG Objectives

22.c.11 Compares and measures Represents and analyzes data

TSG Objectives

22.c.10 Compares and measures Represents and analyzes data Reads and creates scaled picture or bar graphs where each picture/bar represents more than one data point; uses the graph to ask and answer questions.

Classification: Foundations: Similarity Intuiter

Intuitively recognizes objects or situations as similar in some way (objects to suck or not, 2 weeks). Places objects together that are different (6 months) and then alike (12 months).

TSG Objectives

Classification: Hierarchical Classifier

Classifies categories and subcategories using hierarchical inclusion. Conscientiously classifies according to multiple attributes, naming and relating the attributes, understanding that objects could belong to more than one group.

Trajectory Level

TSG Objectives

22.c.8 Compares and measures Represents and analyzes data Organizes, represents, and interprets data with up to four categories; describes data points; asks and answers questions related to the total data set and to its parts

TSG Objectives

22.c.9 Compares and measures Represents and analyzes data

Classification: Multiple Attribute Classifier Classifies objects by multiple attributes in a single sort.

TSG Objectives

22.c.5 Compares and measures Represents and analyzes data

Classification: Similar/Dissimilar Maker By 18 months, forms sets in which objects in each set are identical and objects in the other sets are different, and by 2 years, intuitively forms groups with objects that are similar on some attributes (may be mixed and inconsistent), but not necessarily identical.

TSG Objectives

Classification: Simple Sorter Follows verbal rules for sorting scaffolded by an adult. (These may be made with shifting criteria; nevertheless, they play an essential role in number, through the unitizing process.) Can “fix” a simple sort with mistakes.

TSG Objectives

Classification: Sorter by Similar Attributes Sorts objects according to an explicit attribute (although still may decide to switch attributes during the sorting). The end result may appear to reflect adult categorizations, but often has a different basis, such as general resemblance.

TSG Objectives

Trajectory Level

Comparing Number

Comparing Number: Benchmarks Estimator

Counts a portion of the to-be-estimated collection and uses that as a benchmark from which an estimate is made, intuitively or using repeated addition or multiplication. Scanning can be linked to recalled benchmarks.

TSG Objectives

20.d.8	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write four-digit numbers; multiplies one-digit whole numbers by 10s (10 – 90); rounds three-digit whole numbers to the nearest ten or hundred
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Comparing Number: Comparison Senser: Foundations

From the first months of life, children are sensitive to a change in the number, either of a change of very small collections, such as 1 vs. 2, or large changes in larger collection, such as double the number. Therefore, we know infants have an unconscious, innate sensitivity to such simple equivalence comparisons.

TSG Objectives

Comparing Number: Composition Estimator

Decomposes or partitions the to-be-estimated set into convenient subset sizes, then recomposes the numerosity. Initially, this is done with regular arrangements using repeated addition or multiplication. Later, the process can be done with irregular arrangements and children more consistently use multiplication skills to recompose.

TSG Objectives

20.d.9	Uses number concepts and operations	Understands and uses place value and base ten
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TSG Objectives

20.d.8	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write four-digit numbers; multiplies one-digit whole numbers by 10s (10 – 90); rounds three-digit whole numbers to the nearest ten or hundred
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Comparing Number: Counting Comparer (10)

Compares with counting, even when larger collection's objects are smaller, up to 10.

Trajectory Level

TSG Objectives

Comparing Number: Counting Comparer (5)

Compares with counting, even when larger collection's objects are smaller. Later, figures out how many more or less.

TSG Objectives

Comparing Number: Counting Comparer (Same Size)

Accurately compares via counting, but only when objects are about the same size and groups are small (up to about 5). Not always accurate when the larger collection's objects are smaller in size than the objects in the smaller collection. Accurately counts two equal collections, but, when asked, says the collection of larger blocks has more.

TSG Objectives

20.b.6

Uses number concepts and operations

Quantifies

Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many

Comparing Number: Early Comparer of Dissimilar Items

Matches small, equal collections consisting of different items, showing that they are the same number.

TSG Objectives

Comparing Number: Early Comparer of Similar Items

Compares collections of 1 to 4 items verbally or nonverbally ("just by looking"). The items must be the same. May compare the small collections using number words "two" and "three" (approximately age 3; 2), and "three" and others (age 3; 6). Some do this even before they can accurately by using recognition of number/subitizing for these quantities. May transfer an ordering relation from one pair of collections to another.

TSG Objectives

Comparing Number: First-Second Ordinal Counter

Identifies the "first" and often "second" objects in a sequence.

Trajectory Level

TSG Objectives

22.c.2 Compares and measures Represents and analyzes data Knows a few ordinal numbers

Comparing Number: Many-to-One Corresponder

Recognizes that two very small collections have the “same number” by intuitively making a correspondence between the items in each collection. At this level, in certain situations, children may also put objects, words, or actions in one-to-one or many-to-one correspondence or a mixture.

TSG Objectives

Comparing Number: Matching Comparer

Compares groups of 1–6 by matching.

TSG Objectives

Comparing Number: Mental Number Line to 10

Uses internal images and knowledge of number relationships to determine relative size and position.

TSG Objectives

20.c.5 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.6 Uses number concepts and operations Connects numerals with their Quantities Identifies numerals to 10 by name and connects each to counted objects

TSG Objectives

20.c.7 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.8 Uses number concepts and operations Connects numerals with their Quantities Identifies numerals to 20 by name and connects each to counted objects; represents how many by writing one-digit numerals and some two-digit numerals

Trajectory Level

Comparing Number: Mental Number Line to 100

Uses knowledge of number relationships and mental images, including how ones can be embedded in tens, to determine relative size and position.

TSG Objectives

20.c.15 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.14 Uses number concepts and operations Connects numerals with their Quantities Represents fractional quantities as part of a whole ($a/2$, $a/3$, $a/4$, $a/6$, $a/8$); uses relation symbols ($<$, $>$, $=$) to show fractional comparisons

TSG Objectives

20.d.4 Uses number concepts and operations Understands and uses place value and base ten Uses place-value understanding to represent and write two-digit numbers, add one- and two- digit numbers (within 100), and subtract multiples of 100 from multiples of 10 ($10 - 90$)

TSG Objectives

20.d.3 Uses number concepts and operations Understands and uses place value and base ten

TSG Objectives

20.c.10 Uses number concepts and operations Connects numerals with their Quantities Represents how many by writing one-, two-, and three-digit numerals to 120; uses relational symbols ($<$, $>$, $=$) to indicate relationships between whole numbers

TSG Objectives

20.c.12 Uses number concepts and operations Connects numerals with their Quantities Represents how many by writing one-, two-, three-, and four-digit numerals to 1000; uses relational symbols to compare and order whole numbers

TSG Objectives

20.c.13 Uses number concepts and operations Connects numerals with their Quantities

Trajectory Level

TSG Objectives

20.c.11 Uses number concepts and operations Connects numerals with their Quantities

Comparing Number: Mental Number Line to 1000

Uses internal images and knowledge of number relationships, including place value, to determine relative size and position.

TSG Objectives

20.d.6 Uses number concepts and operations Understands and uses place value and base ten Uses place-value understanding to represent and write three-digit numbers (including expanded form); adds up to four two-digit numbers; adds and subtracts three-digit numbers (within 1,000)

TSG Objectives

20.c.11 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.12 Uses number concepts and operations Connects numerals with their Quantities Represents how many by writing one-, two-, three-, and four-digit numerals to 1000; uses relational symbols to compare and order whole numbers

TSG Objectives

20.c.13 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.14 Uses number concepts and operations Connects numerals with their Quantities Represents fractional quantities as part of a whole ($a/2$, $a/3$, $a/4$, $a/6$, $a/8$); uses relation symbols ($<$, $>$, $=$) to show fractional comparisons

TSG Objectives

20.c.15 Uses number concepts and operations Connects numerals with their Quantities

Trajectory Level

TSG Objectives

20.d.7 Uses number concepts and operations Understands and uses place value and base ten

TSG Objectives

20.c.10 Uses number concepts and operations Connects numerals with their Quantities Represents how many by writing one-, two-, and three-digit numerals to 120; uses relational symbols (<, >, =) to indicate relationships between whole numbers

Comparing Number: Mental Number Line to 5

Uses knowledge of counting number relationships to determine relative size and position when given perceptual support.

TSG Objectives

20.c.2 Uses number concepts and operations Connects numerals with their Quantities Recognizes and names a few numerals

TSG Objectives

20.c.3 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.4 Uses number concepts and operations Connects numerals with their Quantities Identifies numerals to 5 by name and connects each to counted objects

Comparing Number: Object Corresponder

Puts objects into one-to-one correspondence, although may not understand that this creates equal groups (age 2; 8).

TSG Objectives

Comparing Number: One-to-One Object Corresponder

Puts objects into 1-to-1 correspondence when it is clear the materials are a physical “pair.” Implicitly sensitive to the relation of “more than/less than” involving very small numbers (from 1 to 2 years of age). Uses words to include “more,” “less,” or “same.”

TSG Objectives

Trajectory Level

Comparing Number: Ordinal Counter

Identifies and uses ordinal numbers from “first” to “tenth.”

TSG Objectives

22.c.4	Compares and measures	Represents and analyzes data	Creates and reads simple graph; uses simple comparison and ordinal terms to describe findings
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TSG Objectives

22.a.6	Compares and measures	Measures objects	Uses multiples of the same unit to measure; uses numbers to compare; knows the purpose of standard measuring tools
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TSG Objectives

22.a.7	Compares and measures	Measures objects	
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TSG Objectives

22.a.8	Compares and measures	Measures objects	Uses measurement words and some standard measurement tools accurately; uses ordinal numbers from first to tenth
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Comparing Number: Perceptual Comparer

Compares collections that are quite different in number (e.g., one is at least twice the other). Compares similar collections but only involving very small numbers. Compares collections using number words “one” and “two” (age 2; 8).

TSG Objectives

Comparing Number: Place Value Comparer

Compares numbers with place value understandings.

TSG Objectives

20.d.2	Uses number concepts and operations	Understands and uses place value and base ten	Indicates base-ten equivalents for numbers 11 – 19 using objects and drawings; may use simple equations
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TSG Objectives

20.c.9	Uses number concepts and operations	Connects numerals with their Quantities	
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Trajectory Level

TSG Objectives

20.c.10	Uses number concepts and operations	Connects numerals with their Quantities	Represents how many by writing one-, two-, and three-digit numerals to 120; uses relational symbols (<, >, =) to indicate relationships between whole numbers
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Comparing Number: Scanning with Intuitive Quantification Estimator

Scans a group of objects and relates the results to a mental number line to perform a useful numerosity estimation.

TSG Objectives

20.c.13	Uses number concepts and operations	Connects numerals with their Quantities
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TSG Objectives

20.c.14	Uses number concepts and operations	Connects numerals with their Quantities	Represents fractional quantities as part of a whole ($a/2$, $a/3$, $a/4$, $a/6$, $a/8$); uses relation symbols (<, >, =) to show fractional comparisons
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TSG Objectives

20.c.10	Uses number concepts and operations	Connects numerals with their Quantities	Represents how many by writing one-, two-, and three-digit numerals to 120; uses relational symbols (<, >, =) to indicate relationships between whole numbers
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TSG Objectives

20.d.5	Uses number concepts and operations	Understands and uses place value and base ten
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TSG Objectives

20.c.11	Uses number concepts and operations	Connects numerals with their Quantities
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TSG Objectives

20.c.15	Uses number concepts and operations	Connects numerals with their Quantities
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Trajectory Level

TSG Objectives

20.d.6	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write three-digit numbers (including expanded form); adds up to four two-digit numbers; adds and subtracts three-digit numbers (within 1,000)
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TSG Objectives

20.c.12	Uses number concepts and operations	Connects numerals with their Quantities	Represents how many by writing one-, two-, three-, and four-digit numerals to 1000; uses relational symbols to compare and order whole numbers
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Comparing Number: Serial Orderer to 5

Orders quantities (dots) or numerals up to 5. Similarly orders lengths marked into units.

TSG Objectives

Comparing Number: Serial Orderer to 6+

Orders quantities (dots) or numerals to 6 and beyond. Similarly orders lengths marked into units.

TSG Objectives

Comparing Number: Spatial Extent Estimator

Extends sets and number categories to include “small numbers,” which are usually subitized, not estimated; “middle-size numbers” (e.g., 10–20); and “large numbers.” The arrangement of the to-be-estimated set affects the difficulty.

TSG Objectives

Comparing Number: Spatial Extent Estimator—Small/Big

Estimates which set is more or less if the differences are clear (e.g., one is double the other). Names a "small number" (e.g. from 1-4) for sets that cover little space and a "big number" (10-20 or more) for sets that cover a lot of space. Children classify numbers "little"/"big" idiosyncratically, and this may change with the size of the to-be-estimated objects.

TSG Objectives

Trajectory Level

Composing 2D Shapes

Composing 2D Shapes: Picture Maker

Puts several shapes together to make one part of a picture (e.g., two shapes for one arm). Uses trial and error, and does not anticipate creation of new geometric shape. Chooses shapes using “general shape” or side length. Fills “easy” “Pattern Block Puzzles” that suggest the placement of each shape (note that in the example on the right the child is trying to put a square in the puzzle where its right angles will not fit).

TSG Objectives

Composing 2D Shapes: Piece Assembler

Makes pictures in which each shape represents a unique role (e.g., one shape for each body part) and shapes touch. Fills simple puzzles in which all shapes are outlined, often using trial and error.

TSG Objectives

Composing 2D Shapes: Separate Shapes Actor: Foundations

Infants and toddlers manipulate shapes as individuals, but usually do not combine them to compose a larger shape.

TSG Objectives

Composing 2D Shapes: Shape Composer

Composes shapes with anticipation (“I know what will fit!”). Chooses shapes using angles as well as side lengths. Rotation and flipping are used intentionally to select and place shapes. In the “Pattern Block Puzzles” below, all angles are correct, and patterning is evident.

TSG Objectives

Composing 2D Shapes: Shape Composer-Units of Units

Builds and applies units of units (shapes made from other shapes). For example, in constructing spatial patterns, extends patterning activity to create a tiling with a new unit shape—a unit of unit shapes that they recognize and consciously construct.

Trajectory Level

TSG Objectives

21.b.14	Explores and describes spatial relationships and shapes	Understands shapes	Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions
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TSG Objectives

21.b.15	Explores and describes spatial relationships and shapes	Understands shapes	
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TSG Objectives

21.b.13	Explores and describes spatial relationships and shapes	Understands shapes	
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Composing 2D Shapes: Shape Composite Repeater

Constructs and duplicates units of units (shapes made from other shapes) intentionally; understands each as being both multiple small shapes and one larger shape. May continue a pattern of shapes that leads to tiling.

TSG Objectives

21.b.12	Explores and describes spatial relationships and shapes	Understands shapes	Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes
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TSG Objectives

21.b.11	Explores and describes spatial relationships and shapes	Understands shapes	
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Trajectory Level

Composing 2D Shapes: Shape Decomposer (with Help)

Decomposes shapes using imagery that is suggested and supported by the task or environment.

TSG Objectives

21.b.9 Explores and describes spatial relationships and shapes Understands shapes

TSG Objectives

21.b.10 Explores and describes spatial relationships and shapes Understands shapes Distinguishes essential attributes of triangles, rectangles, squares, trapezoids, half circles, and quarter circles; visualizes and creates known shapes

Composing 2D Shapes: Shape Decomposer with Imagery

Decomposes shapes flexibly using independently generated imagery. That is, decomposition is intentionally specified by the child.

TSG Objectives

21.b.12 Explores and describes spatial relationships and shapes Understands shapes Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes

TSG Objectives

21.b.11 Explores and describes spatial relationships and shapes Understands shapes

Composing 2D Shapes: Shape Decomposer with Units of Units

Decomposes shapes flexibly using independently generated imagery and planned decompositions of shapes that themselves are decompositions.

Trajectory Level

TSG Objectives

21.b.14	Explores and describes spatial relationships and shapes	Understands shapes	Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions
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TSG Objectives

21.b.13	Explores and describes spatial relationships and shapes	Understands shapes	
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TSG Objectives

21.b.15	Explores and describes spatial relationships and shapes	Understands shapes	
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Composing 2D Shapes: Simple Decomposer

Decomposes (“takes apart” into smaller shapes) simple shapes that have obvious clues as to their decomposition.

TSG Objectives

Composing 2D Shapes: Substitution Composer

Makes new shapes out of smaller shapes, and uses trial and error to substitute groups of shapes for other shapes to create new shapes in different ways.

TSG Objectives

Trajectory Level

Composing 3D shapes

Composing 3D shapes: Line Maker

Shows use of relationship of “next to” to make a (one-dimensional) line of blocks.

TSG Objectives

Composing 3D shapes: Picture Maker (3D)

Uses multiple spatial relations, extending in multiple directions and with multiple points of contact among components, showing flexibility in integrating parts of the structure. Produces arches, enclosures, corners, and crosses, although may use unsystematic trial and error and simple addition of pieces.

TSG Objectives

Composing 3D shapes: Piece Assembler (3D)

Builds vertical and horizontal components within a building, but within a limited range, such as building a “floor” or a simple “wall.” These, then, are two-dimensional structures.

TSG Objectives

Composing 3D shapes: Same Shape Stacker

Shows use of relationship of “on” to stack congruent blocks, or those that show a similarly helpful relationship to make stacks or lines.

TSG Objectives

Composing 3D shapes: Separate Blocks Actor: Foundations

Either places blocks randomly or manipulates shapes as individuals, but does not combine them to compose a larger shape. May pound, clap together, or use slide blocks or single blocks to represent an object, such as a house or truck.

TSG Objectives

Composing 3D shapes: Shape Composer - Unit of Units (3D)

Makes complex towers or other structures, involving multiple levels with ceilings (fitting the ceilings), and adult-like structures with blocks, including arches and other substructures.

Trajectory Level

TSG Objectives

21.b.13 Explores and describes spatial relationships and shapes Understands shapes

TSG Objectives

21.b.15 Explores and describes spatial relationships and shapes Understands shapes

TSG Objectives

21.b.14 Explores and describes spatial relationships and shapes Understands shapes Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions

Composing 3D shapes: Shape Composer (3D)

Composes shapes with anticipation, understanding what 3D shape will be produced with a composition of 2 or ore other (simple, familiar) 3D shapes. Can produce arches (with vertical interior space), enclosures (with internal horizontal space), corners, and crosses systematically. Builds enclosures and arches several blocks high. Later in this level, children add depth to make 3D structures, and they add roofs across structures multiple blocks high (but they may have no internal spaces).

TSG Objectives

21.b.11 Explores and describes spatial relationships and shapes Understands shapes

Trajectory Level

TSG Objectives

21.b.12	Explores and describes spatial relationships and shapes	Understands shapes	Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes
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Composing 3D shapes: Stacker Shows use of the spatial relationship of “on” to stack blocks, although choice of blocks may be unsystematic.

TSG Objectives

Composing 3D shapes: Substitution Composer and Shape Composite Repeater (3D) Substitutes a composite for a congruent whole. Builds complex bridges with multiple arches, ramps and stairs at the ends. Structures are 3D, often including roofs and multiple internal spaces.

TSG Objectives

21.b.15	Explores and describes spatial relationships and shapes	Understands shapes
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TSG Objectives

21.b.14	Explores and describes spatial relationships and shapes	Understands shapes	Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions
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TSG Objectives

21.b.12	Explores and describes spatial relationships and shapes	Understands shapes	Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes
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Trajectory Level

TSG Objectives

21.b.13	Explores and describes spatial relationships and shapes	Understands shapes
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TSG Objectives

21.b.11	Explores and describes spatial relationships and shapes	Understands shapes
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Trajectory Level

Composing Numbers

Composing Numbers: Actor on Parts: Foundations

Displays actions that show intuition about parts and wholes such as gathering objects together. Only nonverbally recognizes parts and wholes. Recognizes that sets can be combined in different orders but may not explicitly recognize that groups are additively composed of smaller groups.

TSG Objectives

Composing Numbers: Composer to 10

Knows number combinations to totals of 10. Quickly names parts of any whole, or the whole given parts. Doubles to 20.

TSG Objectives

22.b.9 Compares and Measures Time and Money
measures

TSG Objectives

22.b.8 Compares and Measures Time and Money Tells and writes time in hours and half-hours using both analog
measures and digital clocks; makes amounts using pennies (P), nickels (N), and dimes (D)

TSG Objectives

20.b.6 Uses number Quantifies Makes sets of 6-10 objects and then describes the parts;
concepts and operations identifies which part has more, less, or the same (equal); counts
all or counts on to find out how many

TSG Objectives

22.b.7 Compares and Measures Time and Money
measures

TSG Objectives

20.f.4 Uses number Applies number combinations Adds and subtracts while numbers fluently within 10 using
concepts and operations and mental number strategies in mental strategies (counting on, making ten,
operations mathematical operations decomposition/recomposing, addition/subtraction relationship,
and easier equivalent known sums)

Composing Numbers: Composer to 4, then 5

Knows number combinations. Quickly names parts of any whole, or the whole given the parts.

Trajectory Level

TSG Objectives

20.b.4	Uses number concepts and operations	Quantifies	Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts
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TSG Objectives

20.f.2	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts whole numbers fluently within five
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TSG Objectives

20.b.5	Uses number concepts and operations	Quantifies	
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Composing Numbers: Composer to 7 Knows number combinations to totals of 7. Quickly names parts of any whole, or the whole given parts. Doubles to 10.

TSG Objectives

20.b.6	Uses number concepts and operations	Quantifies	Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many
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TSG Objectives

20.f.3	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	
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Composing Numbers: Composer with Tens and Ones Understands 2-digit numbers as tens and ones; count with dimes and pennies; 2-digit addition with regrouping.

TSG Objectives

20.f.5	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	
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TSG Objectives

22.b.7	Compares and measures	Measures Time and Money	
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Trajectory Level

TSG Objectives

22.b.9 Compares and measures Measures Time and Money

TSG Objectives

20.f.6 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts while numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory

TSG Objectives

22.b.8 Compares and measures Measures Time and Money Tells and writes time in hours and half-hours using both analog and digital clocks; makes amounts using pennies (P), nickels (N), and dimes (D)

Composing Numbers: Deriver +/- Uses flexible strategies and derived combinations (e.g., “7 + 7 is 14, so 7 + 8 is 15) to solve all types of problems. Includes Break Apart to Make Ten (BAMT). Can simultaneously think of 3 numbers within a sum, and can move part of a number to another, aware of the increase in one and the decrease in another. Solves simple cases of multidigit addition (and, often, subtraction) by incrementing tens and/or ones.

TSG Objectives

20.f.6 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts while numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory

TSG Objectives

22.b.10 Compares and measures Measures Time and Money Tells and writes time to the nearest five minutes; indicates a.m. and p.m.; solves words problems involving coins (P, N, D, Q) and dollar bills, and expresses the answer using currency symbols

TSG Objectives

22.b.9 Compares and measures Measures Time and Money

Composing Numbers: Inexact Part-Whole Recognizer Knows that a whole is bigger than parts, but may not accurately quantify (label with numbers). (May show intuitive knowledge of commutativity, and, later, associativity with physical groups, later in more abstract contexts, including numbers.)

Trajectory Level

TSG Objectives

Composing Numbers: Multidigit +/-

Uses composition of tens and all previous strategies to solve multidigit +/- problems.

TSG Objectives

22.b.13 Compares and measures Measures Time and Money

TSG Objectives

20.f.8 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts while numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on

TSG Objectives

20.f.9 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations

TSG Objectives

22.b.11 Compares and measures Measures Time and Money

TSG Objectives

22.b.10 Compares and measures Measures Time and Money Tells and writes time to the nearest five minutes; indicates a.m. and p.m.; solves words problems involving coins (P, N, D, Q) and dollar bills, and expresses the answer using currency symbols

TSG Objectives

22.b.12 Compares and measures Measures Time and Money Solves one-step word problems related to time to the nearest minute

Composing Numbers: Parts Combiner

Recognizes that sets can be combined in different orders, but may not explicitly recognize that groups are additively composed of smaller groups. The toddler also recognizes part-whole relations in nonverbal, intuitive, perceptual situations and can nonverbally represent parts that make a whole.

Trajectory Level

TSG Objectives

Composing Numbers: Problem Solver +/-

Solves all types of problems, with flexible strategies and known combinations. Multidigit may be solved by incrementing or combining tens and ones (latter not used for join, change unknown).

TSG Objectives

22.b.12	Compares and measures	Measures Time and Money	Solves one-step word problems related to time to the nearest minute
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TSG Objectives

20.f.8	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on
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TSG Objectives

20.f.7	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations
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TSG Objectives

22.b.13	Compares and measures	Measures Time and Money
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TSG Objectives

22.b.10	Compares and measures	Measures Time and Money	Tells and writes time to the nearest five minutes; indicates a.m. and p.m.; solves words problems involving coins (P, N, D, Q) and dollar bills, and expresses the answer using currency symbols
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TSG Objectives

22.b.11	Compares and measures	Measures Time and Money
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Trajectory Level

Counting

Counting: Chanter

Chants number words in “sing-song” fashion and may run them together. The number words may be indistinguishable from one another (‘onetwothree’). May begin a nonverbal object “counting” such as copying an adult’s item-by-item placement of objects.

TSG Objectives

20.a.2	Uses number concepts and operations	Counts	Verbally counts (not always in the correct order)
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Counting: Corresponder

Keeps one-to-one correspondence between counting words and objects (one word for each object), at least for small groups of objects laid in a line.

TSG Objectives

20.a.4	Uses number concepts and operations	Counts	Verbally counts to 10; counts up to five objects accurately, using one number name for each object
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Counting: Counter (10)

Counts arrangements of objects to 10 with understanding of the cardinal principle. May be able to read and write numerals to represent 1–10. May be able to tell the number just after or just before another number, but only by counting up from 1. Verbal counting to 20 is developing.

TSG Objectives

20.b.6	Uses number concepts and operations	Quantifies	Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many
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TSG Objectives

20.a.6	Uses number concepts and operations	Counts	Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting
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Counting: Counter (Small Numbers)

Accurately counts objects in a line to 5 and answers the “how many” question with the last number counted, understanding that this represents the total number of objects (the cardinal principle).

Trajectory Level

TSG Objectives

20.a.5 Uses number concepts and operations Counts

Counting: Counter and Producer (10+)

Counts and counts out objects accurately to 10, then beyond (to about 30). Has explicit understanding of cardinality (how numbers tell how many). Keeps track of objects that have and have not been counted, even in different arrangements. Writes or draws to represent 1 to 10 (then, 20, then 30). Gives next number (usually to 20s or 30s). Separates the decade and the ones part of a number word and begins to relate each part of a number word/numeral to the quantity to which it refers. Recognizes errors in others' counting and can eliminate most errors in own counting (point-object) if asked to try hard.

TSG Objectives

20.a.8 Uses number concepts and operations Counts

Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20

TSG Objectives

20.a.7 Uses number concepts and operations Counts

TSG Objectives

20.a.6 Uses number concepts and operations Counts

Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting

TSG Objectives

20.b.6 Uses number concepts and operations Quantifies

Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many

Counting: Counter Backward from 10

Counts backward from 10 to 1, verbally, or when removing objects from a group. "10, 9, 8, 7, 6, 5, 4, 3, 2, 1!"

Trajectory Level

TSG Objectives

20.a.8	Uses number concepts and operations	Counts	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20
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Counting: Counter Beyond 100 Accurately counts beyond 100, recognizing the patterns of ones, tens, and hundreds.

TSG Objectives

20.d.5	Uses number concepts and operations	Understands and uses place value and base ten
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TSG Objectives

20.a.11	Uses number concepts and operations	Counts
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TSG Objectives

20.d.6	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write three-digit numbers (including expanded form); adds up to four two-digit numbers; adds and subtracts three-digit numbers (within 1,000)
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TSG Objectives

20.a.12	Uses number concepts and operations	Counts	Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts
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Counting: Counter Forward and Back Counts “counting words” (single sequence or skip counts) in either direction. Recognizes that decades sequence mirrors single digit sequence. Switches between sequence and composition views of multidigit numbers easily.

TSG Objectives

20.a.13	Uses number concepts and operations	Counts
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Trajectory Level

TSG Objectives

20.d.4	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write two-digit numbers, add one- and two- digit numbers (within 100), and subtract multiples of 100 from multiples of 10 (10 – 90)
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TSG Objectives

20.a.12	Uses number concepts and operations	Counts	Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts
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TSG Objectives

20.d.3	Uses number concepts and operations	Understands and uses place value and base ten
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Counting: Counter from N (N + 1, N -1)

Counts verbally and with objects from numbers other than 1 (but does not yet keep track of the number of counts). Immediately determines numbers just after or just before.

TSG Objectives

20.a.8	Uses number concepts and operations	Counts	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20
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Counting: Counter of Quantitative Units/Place Value

Understands the base ten numeration system and place value concepts, including ideas of counting in units and multiples of hundreds, tens, and ones. When counting groups of ten, can decompose into 10 ones if that is useful. Understands value of a digit according to the place of the digit within a number. Counts unusual units, such as “wholes” when shown combinations of wholes and parts.

TSG Objectives

20.d.4	Uses number concepts and operations	Understands and uses place value and base ten	Uses place-value understanding to represent and write two-digit numbers, add one- and two- digit numbers (within 100), and subtract multiples of 100 from multiples of 10 (10 – 90)
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TSG Objectives

23.15	Demonstrates knowledge of patterns
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Trajectory Level

TSG Objectives

20.d.3 Uses number concepts and operations Understands and uses place value and base ten

TSG Objectives

23.10 Demonstrates knowledge of patterns Notices more complex patterns in numbers; identifies the core unit of patterns; represents patterns using numerical and letter symbols

TSG Objectives

23.12 Demonstrates knowledge of patterns Uses number patterns to count and to solve problems; uses and explains patterns in counting and addition

TSG Objectives

23.11 Demonstrates knowledge of patterns

TSG Objectives

20.a.10 Uses number concepts and operations Counts Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20

TSG Objectives

20.a.11 Uses number concepts and operations Counts

TSG Objectives

23.14 Demonstrates knowledge of patterns Recognizes arithmetic patterns and explains them using properties of operations

TSG Objectives

23.13 Demonstrates knowledge of patterns

Trajectory Level

TSG Objectives

23.9 Demonstrates knowledge of patterns

Counting: Counter on Keeping Track

Counts forward or back from a given number keeping track of counting acts numerically, first using objects, then by “counting counts.”

TSG Objectives

20.a.10 Uses number concepts and operations Counts Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20

TSG Objectives

20.b.8 Uses number concepts and operations Quantifies Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts

TSG Objectives

20.b.7 Uses number concepts and operations Quantifies

TSG Objectives

20.a.11 Uses number concepts and operations Counts

TSG Objectives

20.b.9 Uses number concepts and operations Quantifies

Counting: Counter on Using Patterns

Keeps track of counting acts, but only by using numerical patterns (spatial, auditory, or rhythmic) for adding 1 to about 3.

TSG Objectives

20.a.10 Uses number concepts and operations Counts Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20

Trajectory Level

TSG Objectives

20.b.7 Uses number concepts and operations Quantifies

TSG Objectives

23.10 Demonstrates knowledge of patterns Notices more complex patterns in numbers; identifies the core unit of patterns; represents patterns using numerical and letter symbols

TSG Objectives

20.b.8 Uses number concepts and operations Quantifies Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts

TSG Objectives

20.b.9 Uses number concepts and operations Quantifies

TSG Objectives

23.11 Demonstrates knowledge of patterns

TSG Objectives

23.7 Demonstrates knowledge of patterns

TSG Objectives

23.8 Demonstrates knowledge of patterns Recognizes, creates, and explains more complex repeating and simple growing patterns

TSG Objectives

23.9 Demonstrates knowledge of patterns

Learning Trajectories: Alignment to Teaching Strategies GOLD



Trajectory Level

Counting: Counter to 100

Counts to 100. Makes decade transitions (e.g., from 29 to 30) starting at any number. "... 78, 79 ... 80, 81 ..."

TSG Objectives

20.a.9 Uses number concepts and operations Counts

TSG Objectives

20.a.14 Uses number concepts and operations Counts

Counts to more than 1,000 using number word patterns (e.g., tens, teens) and skip counting; uses skip counting by 2s, 4s, 5s, 6s, 10s, and 100s

TSG Objectives

20.a.15 Uses number concepts and operations Counts

TSG Objectives

20.d.2 Uses number concepts and operations Understands and uses place value and base ten

Indicates base-ten equivalents for numbers 11 – 19 using objects and drawings; may use simple equations

TSG Objectives

20.a.8 Uses number concepts and operations Counts

Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20

Counting: Number Conserver

Consistently conserves number (i.e., believes number has been unchanged), even in face of perceptual distractions such as the spreading out of objects in a collection.

TSG Objectives

20.a.12 Uses number concepts and operations Counts

Counts to 1,000 to determine how many; uses skip counting (2s, 5s, 10s, and 100s); begins counting forward at any number between 1 and 1000; switches between skip counts

Counting: Number Word Sayer: Foundations

No verbal counting. Names some number words with no sequence.

Trajectory Level

TSG Objectives

20.a.0 Uses number concepts and operations Counts Not Yet

TSG Objectives

20.a.1 Uses number concepts and operations Counts

Counting: Producer (Small Numbers)

Counts out objects to 5. Recognizes that counting is relevant to situations in which a certain number must be placed. Produces a group of 4 objects.

TSG Objectives

20.a.6 Uses number concepts and operations Counts Verbally counts to 20; counts 10-20 objects accurately; knows the last number states how many in all; tells what number (1-10) comes next in order by counting

Counting: Reciter

Verbally counts with distinct words, not necessarily in the correct order above “five.” May say “One, two, three, four, five, seven.” If knows more number words than number of objects, rattles them off quickly at the end; if more objects, “recycles” number words (inflexible list exhaustion)

TSG Objectives

20.a.3 Uses number concepts and operations Counts

Counting: Reciter (10)

Verbally counts to ten with some correspondence with objects, but may either continue an overly rigid correspondence or exhibit performance errors (e.g., skipping, double-counting).

TSG Objectives

20.a.4 Uses number concepts and operations Counts Verbally counts to 10; counts up to five objects accurately, using one number name for each object

Counting: Skip Counter

Counts by fives and twos with understanding.

Trajectory Level

TSG Objectives

20.a.10	Uses number concepts and operations	Counts	Counts to 120 to determine how many; uses skip counting by 2s, 5s, and 10s; begins counting forward at any number between 1 and 120; counts backwards from 20
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Counting: Skip Counter by 10s to 100

Skip counts by tens up to 100 or beyond with understanding; e.g., “sees” groups of 10 within a quantity and counts those groups by 10 (this relates to multiplication and algebraic thinking). “10, 20, 30 ... 100.”

TSG Objectives

20.a.8	Uses number concepts and operations	Counts	Uses number names while counting to 100; counts 30 objects accurately; tells what number comes before and after a specified number up to 20
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Trajectory Level

Disembedding Shapes

Disembedding shapes: Complete Disembedder

Successfully identifies all varieties of complex arrangements.

TSG Objectives

Disembedding shapes: Intuitive Disembedder: Foundations

Can remember and reproduce only one or a small collection of non-overlapping (isolated) shapes.

TSG Objectives

Disembedding shapes: Secondary Structure Disembedder

Identifies embedded figures even when they do not coincide with any primary structures of the complex figure.

TSG Objectives

Disembedding shapes: Shapes in Shapes Disembedder

Identifies shapes embedded within other shapes, such as concentric circles and/or a circle in a square. Identifies primary structures in complex figures.

TSG Objectives

Disembedding shapes: Simple Disembedder

Identifies frame of complex figure. Finds some shapes in arrangements in which figures overlap, but not in those in which figures are embedded within others.

TSG Objectives

20.b.14

Uses number concepts and operations

Quantifies

Compares fractions and explains them using physical models, pictorial representations, and number lines

TSG Objectives

20.b.9

Uses number concepts and operations

Quantifies

Trajectory Level

TSG Objectives

20.b.14	Uses number concepts and operations	Quantifies	Compares fractions and explains them using physical models, pictorial representations, and number lines
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TSG Objectives

20.b.10	Uses number concepts and operations	Quantifies	Answers how much questions about wholes portioned into equal size shares of two and four; verbally labels each part and describes its relationship to the whole
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TSG Objectives

20.b.11	Uses number concepts and operations	Quantifies	
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TSG Objectives

20.b.12	Uses number concepts and operations	Quantifies	Answers how much questions about wholes portioned into equal size shares of two and four; verbally labels each part and describes its relationship to the whole
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TSG Objectives

20.b.13	Uses number concepts and operations	Quantifies	
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TSG Objectives

20.b.14	Uses number concepts and operations	Quantifies	Compares fractions and explains them using physical models, pictorial representations, and number lines
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TSG Objectives

20.b.15	Uses number concepts and operations	Quantifies	
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Trajectory Level

TSG Objectives

21.b.13 Explores and describes spatial relationships and shapes Understands shapes

TSG Objectives

21.b.14 Explores and describes spatial relationships and shapes Understands shapes Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions

TSG Objectives

20.c.14 Uses number concepts and operations Connects numerals with their Quantities Represents fractional quantities as part of a whole ($a/2$, $a/3$, $a/4$, $a/6$, $a/8$); uses relation symbols ($<$, $>$, $=$) to show fractional comparisons

TSG Objectives

20.c.15 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.c.14 Uses number concepts and operations Connects numerals with their Quantities Represents fractional quantities as part of a whole ($a/2$, $a/3$, $a/4$, $a/6$, $a/8$); uses relation symbols ($<$, $>$, $=$) to show fractional comparisons

TSG Objectives

21.b.13 Explores and describes spatial relationships and shapes Understands shapes

Trajectory Level

TSG Objectives

21.b.15 Explores and describes spatial relationships and shapes Understands shapes

TSG Objectives

20.c.14 Uses number concepts and operations Connects numerals with their Quantities Represents fractional quantities as part of a whole ($\frac{a}{2}$, $\frac{a}{3}$, $\frac{a}{4}$, $\frac{a}{6}$, $\frac{a}{8}$); uses relation symbols ($<$, $>$, $=$) to show fractional comparisons

TSG Objectives

21.b.14 Explores and describes spatial relationships and shapes Understands shapes Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions

TSG Objectives

20.b.8 Uses number concepts and operations Quantifies Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts

TSG Objectives

20.c.13 Uses number concepts and operations Connects numerals with their Quantities

TSG Objectives

20.b.8 Uses number concepts and operations Quantifies Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts

TSG Objectives

20.b.10 Uses number concepts and operations Quantifies Answers how much questions about wholes portioned into equal size shares of two and four; verbally labels each part and describes its relationship to the whole



Trajectory Level

TSG Objectives

21.b.15	Explores and describes spatial relationships and shapes	Understands shapes
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Trajectory Level

Length

Length: Conceptual Ruler Measurer

Possesses an “internal” measurement tool. Mentally moves along an object, segmenting it and counting the segments. Operates arithmetically on measures (“connected lengths”). Subdivides a unit at least into halves. Estimates with accuracy.

TSG Objectives

22.a.15 Compares and measures Measures objects

TSG Objectives

22.a.12 Compares and measures Measures objects Measures and compares the length of two objects using standard length units

TSG Objectives

22.a.13 Compares and measures Measures objects

TSG Objectives

22.a.14 Compares and measures Measures objects Solves one-step word problems related to measurement of liquid volume, mass, area, and perimeter.

Length: End-to-End Length Measurer

Lays units end to end. May not recognize the need for equal-length units or be able to measure if there are fewer units that needed. The ability to apply resulting measures to comparison situations develops later in this level. (This develops in parallel with “Serial Orderer to 5 (Length)”).

TSG Objectives

22.a.6 Compares and measures Measures objects Uses multiples of the same unit to measure; uses numbers to compare; knows the purpose of standard measuring tools

TSG Objectives

22.a.7 Compares and measures Measures objects

TSG Objectives

22.a.8 Compares and measures Measures objects Uses measurement words and some standard measurement tools accurately; uses ordinal numbers from first to tenth

Trajectory Level

Length: Length Direct Comparer Physically aligns two objects to determine which is longer or if they are the same length. Uses terms: long, longer, longest.

TSG Objectives

22.a.4	Compares and measures	Measures objects	Compares and orders a small set of objects as appropriate according to size, length, weight, area, or volume.
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TSG Objectives

22.a.3	Compares and measures	Measures objects
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Length: Length Indirect Comparer Compares the length of two objects by representing them with a third object. Uses terms: long, longer, longest, short, shorter, shortest. When asked to measure, may assign a length by guessing or moving along a length while counting (without equal-length units). May be able to measure with a ruler, but often lacks understanding or skill (e.g., ignores starting point).

TSG Objectives

22.a.4	Compares and measures	Measures objects	Compares and orders a small set of objects as appropriate according to size, length, weight, area, or volume.
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Length: Length Measurer Considers the length of a bent path as the sum of its parts (not the distance between the endpoints). Measures, knowing need for identical units, relationship between different units, partitions of unit, zero point on rulers, and accumulation of distance. Begins to estimate.

TSG Objectives

22.a.11	Compares and measures	Measures objects
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TSG Objectives

22.a.12	Compares and measures	Measures objects	Measures and compares the length of two objects using standard length units
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Length: Length Quantity Recognizer Identifies length/distance as an attribute. May understand length as an absolute descriptor (e.g., all adults are tall), but not as a comparative (e.g., one person is taller than another). May compare non-corresponding parts of shape in determining side length.

Trajectory Level

TSG Objectives

22.a.2 Compares and Measures objects Makes simple comparisons between two objects
measures

TSG Objectives

22.a.3 Compares and Measures objects
measures

Length: Length Senser: Foundations Makes simple comparisons of length intuitively (similar to what we saw in Subitizing) as young as six months of age. However, may not recognize length as a distinct a

TSG Objectives

22.a.1 Compares and Measures objects
measures

Length: Length Unit Relater and Repeater Measures by repeating (iterating) a single unit and understands the need for equal-length unit. Relates the size and number of units (inverse relationship). Can add two lengths to obtain the length of a whole. Often can use rulers with minimal guidance in straightforward situations.

TSG Objectives

22.a.11 Compares and Measures objects
measures

TSG Objectives

22.a.9 Compares and Measures objects
measures

TSG Objectives

22.a.10 Compares and Measures objects Measures length accurately and expresses the measurement in
measures whole numbers

Length: Serial Orderer to 5 Orders lengths, marked in 1 to 5 units. Also, can compare unmarked lengths that are clearly different using broad categories (“big” and “small”) and so can order 3 to 5 such objects but only by trial-and-error. With an increase in working memory, begins to build a mental image of the final ordering in which the lengths increase “bit by bit” with each successive length the smallest increase. This leads to more accurate and somewhat more efficient ordering. (This level develops in parallel with “End-to-End Length Measurer”.)

Trajectory Level

TSG Objectives

Length: Serial Orderer to 6+

Orders lengths, marked in 1 to 6 units. Understands at least intuitively that any set of objects of different lengths can be placed into a series that always increases (or decreases) in length, so spontaneously seriates with few errors by selecting the shortest (or longest) object, then the next shortest (the one with the “smallest difference”), and so forth.

TSG Objectives

22.a.5 Compares and Measures objects
 measures

Trajectory Level

Multiplying / Dividing

Multiplying/Dividing: Beginning Grouper and Distributive Sharer

Makes small groups (fewer than 5). Shares by “dealing out,” but usually only between 2 people. May not appreciate the numerical result.

TSG Objectives

Multiplying/Dividing: Concrete Modeler \times/\div

Solves small-number multiplying problems by grouping – making each group and counting all. Solves division/sharing problems with informal strategies, using concrete objects; up to 20 objects and 2-5 people. May not understand equivalence of groups.

TSG Objectives

20.b.8	Uses number concepts and operations	Quantifies	Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts
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TSG Objectives

20.b.7	Uses number concepts and operations	Quantifies
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TSG Objectives

20.b.9	Uses number concepts and operations	Quantifies
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Multiplying/Dividing: Deriver \times/\div

Uses strategies, patterns, de/composition ($12 \times 2 = 10 \times 2 + 2 + 2$) and derived combinations, such as multiplying $\times 9$ as $10 - 1$ or 7×8 from $7 \times 7 + 7$. Solves multidigit problems by operating on tens and ones separately.

TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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Trajectory Level

TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental)
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TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.f.6	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory
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Multiplying/Dividing: Grouper and Distributive Sharer

Makes small equal groups (fewer than 6). Deals out equally between two or more recipients, but may not understand that equal quantities are produced.

TSG Objectives

20.b.8	Uses number concepts and operations	Quantifies	Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts
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TSG Objectives

20.b.9	Uses number concepts and operations	Quantifies
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TSG Objectives

20.b.7	Uses number concepts and operations	Quantifies
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Multiplying/Dividing: Multidigit X/÷

(GONE) Uses multiple strategies, from compensating to paper-and-pencil procedures.

Trajectory Level

TSG Objectives

20.f.8	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on
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TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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TSG Objectives

20.f.9	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	
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TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental
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TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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Multiplying/Dividing: Nonquantitative Sharer: Foundations

Gives some, but not necessarily an equal number to each person.

TSG Objectives

Multiplying/Dividing: Partitive Divisor

Figures out how many are in each group. May first repeatedly add a divisor until the dividend is reached.

Trajectory Level

TSG Objectives

20.e.7 Uses number concepts and operations Applies properties of mathematical operations and relationships

TSG Objectives

20.f.8 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts whole numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on

TSG Objectives

20.f.7 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations

TSG Objectives

20.e.8 Uses number concepts and operations Applies properties of mathematical operations and relationships Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental

TSG Objectives

20.e.9 Uses number concepts and operations Applies properties of mathematical operations and relationships

Multiplying/Dividing: Parts and Whole X/\div

Understands the inverse relation between divisor and quotient in simple, concrete situations.

TSG Objectives

Multiplying/Dividing: Problem Solver X/\div

Solves many types of multiplicative problems, with flexible strategies and known combinations. Multidigit may be solved using combinations separately on ones and tens.

Trajectory Level

TSG Objectives

20.f.8	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on
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TSG Objectives

20.f.7	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	
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TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental
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TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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Multiplying/Dividing: Skip Counter X/÷

Uses repeated adding, additive doubling, or skip counting to solve multiplication and for measurement division (finding out how many groups) problems. Uses trial and error for partitive division (finding out how many in each group).

TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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Trajectory Level

TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.e.6	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves one- and two- step word problems of various types using addition and subtraction (within 100) and explains strategies; uses repeated addition to find the number of objects presented in rectangular arrays (up to five rows and five columns)
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TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental)
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TSG Objectives

20.f.6	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory
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TSG Objectives

20.e.5	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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Trajectory Level

Patterning

Patterning: Beginning Arithmetic Patterner

Recognizes and uses arithmetic patterns with perceptual or pedagogical support, often first those that involve properties of zero. The child also accepts number sentences not in the form of $3 + 4 = 7$ (e.g., $7 = 3 + 4$, or even $3 + 4 = 2 + 5$). This represents a move from an “equals-as-an answer” notion to recognizing that equals means equivalent numbers. In functional thinking, builds two sets (e.g., in a t-chart) following two separate general rules, level 3.

TSG Objectives

20.e.5	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.e.4	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves three-number word problems with answers within 20 using addition properties (associative, communicative, and additive); solves addition and subtraction equations of different types with unknowns in various positions for amounts up to 20
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TSG Objectives

23.12	Demonstrates knowledge of patterns	Uses number patterns to count and to solve problems; uses and explains patterns in counting and addition
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Patterning: Intuitive Patterner: Foundations

Detects and uses patterning implicitly and intuitively, such as in movement activities or common nursery rhymes that repeat words and action. May be attentive to repeating patterns without recognizing them explicitly or accurately, often attending to individual attributes such as color.

TSG Objectives

23.1	Demonstrates knowledge of patterns
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Patterning: Numeric Patterner

Describes a pattern numerically, can translate between geometric and numeric representation of a series. In functional thinking, builds and perceives a t-chart as a sequence of particular instances, level 2.

Trajectory Level

TSG Objectives

23.11 Demonstrates knowledge of patterns

TSG Objectives

23.9 Demonstrates knowledge of patterns

TSG Objectives

23.10 Demonstrates knowledge of patterns

Notices more complex patterns in numbers; identifies the core unit of patterns; represents patterns using numerical and letter symbols

Patterning: Pattern Recognizer

Recognizes a simple pattern, usually ABABAB, as a pattern, even if doesn't yet name or describe it.

TSG Objectives

23.3 Demonstrates knowledge of patterns

TSG Objectives

23.2 Demonstrates knowledge of patterns

Shows interest in simple patterns in everyday life

Patterning: Pattern Translator and Unit Recognizer

Translates patterns into new media or using new materials; that is, abstract and generalize the pattern. Identifies the smallest core unit of a repeating pattern. (Most research indicates this develops later, Miller et al., 2016.) In functional thinking situations (e.g., p. #), does not yet see math relationships in sets of data, level 1.

TSG Objectives

23.8 Demonstrates knowledge of patterns

Recognizes, creates, and explains more complex repeating and simple growing patterns

Trajectory Level

TSG Objectives

23.7 Demonstrates knowledge of patterns

Patterning: Patterner

Recognizes, describes, and builds repeating patterns, including AB but also patterns with core units such as AAB, ABC, and AABC.

TSG Objectives

23.5 Demonstrates knowledge of patterns

TSG Objectives

23.4 Demonstrates knowledge of patterns

Copies simple repeating patterns

TSG Objectives

23.6 Demonstrates knowledge of patterns

Extends and creates simple repeating patterns

Patterning: Patterner AB

Recognizes, describes, and builds repeating ABAB patterns. These involve the following, which many children learn in this order, although this can vary by the task.¹

Fixes AB: Fills in missing element of an ABAB pattern.

Duplicates AB: Duplicates ABABAB pattern (at first may have to work close to the model pattern, but eventually can build the same pattern away from the model pattern or when the model is out of sight).

Extends AB: Extends AB patterns to add multiple units to the end of the pattern. This is easier for children if the pattern ends with a complete unit, but they eventually learn to extend those that end with a partial unit.

TSG Objectives

23.4 Demonstrates knowledge of patterns

Copies simple repeating patterns

Trajectory Level

Patterning: Relational Thinker +/- Recognizes and uses patterns that involve addition and subtraction and, understanding equality, can compare two sides of a number sentence with reasoning, and thus does not have to carry out computations. In functional thinking, creates functional relationships between two data sets but only for specific cases (Blanton et al., 2015)level 4. May use letters to represent numbers, but only as representing objects or fixed values.

TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.e.8	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves, represents, and explain two-step word problems of various types (equal sized groups, arrays, measurement quantities) using properties of whole number operations and multiplication/division inverse relationships; uses estimation strategies (mental
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TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

20.e.4	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves three-number word problems with answers within 20 using addition properties (associative, communicative, and additive); solves addition and subtraction equations of different types with unknowns in various positions for amounts up to 20
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TSG Objectives

20.e.5	Uses number concepts and operations	Applies properties of mathematical operations and relationships
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TSG Objectives

23.13	Demonstrates knowledge of patterns
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Trajectory Level

TSG Objectives

20.e.6	Uses number concepts and operations	Applies properties of mathematical operations and relationships	Solves one- and two- step word problems of various types using addition and subtraction (within 100) and explains strategies; uses repeated addition to find the number of objects presented in rectangular arrays (up to five rows and five columns)
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TSG Objectives

23.12	Demonstrates knowledge of patterns		Uses number patterns to count and to solve problems; uses and explains patterns in counting and addition
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Patterning: Relational Thinker with Multiplication

Recognizes and uses patterns that involve multiplication as repeated addition and use of the distributive property to partition number facts. In functional thinking, generalizes functional relationships between two data sets, (Blanton et al., 2015). level 7. Uses letters as variables represent this relationship

TSG Objectives

23.15	Demonstrates knowledge of patterns		
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TSG Objectives

20.e.9	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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TSG Objectives

23.14	Demonstrates knowledge of patterns		Recognizes arithmetic patterns and explains them using properties of operations
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TSG Objectives

20.e.7	Uses number concepts and operations	Applies properties of mathematical operations and relationships	
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Trajectory Level

TSG Objectives

23.15 Demonstrates knowledge of patterns

Patterning: Relational Thinker-Symbolic +/-

Recognizes and uses patterns that involve addition and subtraction and an understanding of equality. Can compare two sides of a number sentence with reasoning, even when the quantities are represented by variables, such as $a + b = b + a$. In functional thinking, generalizes functional relationships between two data sets, at first just noticing, and later a quantitative relationship (Blanton et al., 2015). level 5, 6 Uses letters for unknown numbers, an initial algebraic notion.

TSG Objectives

23.14 Demonstrates knowledge of patterns

Recognizes arithmetic patterns and explains them using properties of operations

Trajectory Level

Shapes

Shapes: "Same Thing" Comparer: Foundations

Compares real-world objects

TSG Objectives

21.b.1	Explores and describes spatial relationships and shapes	Understands shapes
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Shapes: Angle Recognizer—More Contexts

Recognizes and describe contexts in which angle knowledge is relevant, including corners (can discuss “sharper” angles), crossings (e.g., a pair of scissors), and, later, bent objects and bends (sometimes bends in paths and slopes). Only later can explicitly understand how angle concepts relate to these contexts (e.g., initially may not think of bends in roads as angles; may not be able to add horizontal or vertical to complete the angle in slope contexts; may even see corners as more or less “sharp” without representing the lines that constitute them). Often does not relate these contexts and may represent only some features of angles in each (e.g., oblique line for a ramp in a slope context).

TSG Objectives

21.b.10	Explores and describes spatial relationships and shapes	Understands shapes	Distinguishes essential attributes of triangles, rectangles, squares, trapezoids, half circles, and quarter circles; visualizes and creates known shapes
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Shapes: Angle Representer

Represents various angle contexts as two lines, explicitly including the reference line (horizontal or vertical for slope; a “line of sight” for turn contexts) and, at least implicitly, the size of the angle as the rotation between these lines (may still maintain misconceptions about angle measure, such as relating angle size to the length of side’s distance between endpoints, and may not apply these understandings to multiple contexts).

Trajectory Level

TSG Objectives

21.b.12	Explores and describes spatial relationships and shapes	Understands shapes	Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes
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Shapes: Angle Synthesizer

Combines various meanings of angle (turn, corner, slant), including angle measure.

TSG Objectives

21.b.15	Explores and describes spatial relationships and shapes	Understands shapes	
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TSG Objectives

21.b.14	Explores and describes spatial relationships and shapes	Understands shapes	Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions
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Shapes: Congruence Representer

Refers to geometric properties and explains with transformations.

TSG Objectives

Shapes: Congruence Superposer

Moves and places objects on top of each other to determine congruence. Can also determine congruence by comparing all attributes and all spatial relationships.

TSG Objectives

Shapes: Constructor of Shapes From Parts Exact

Uses manipulatives representing parts of shapes, such as sides and angle “connectors,” to make a shape that is completely correct, based on knowledge of components and properties — relationships between the components.

Trajectory Level

TSG Objectives

21.b.12	Explores and describes spatial relationships and shapes	Understands shapes	Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes
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Shapes: Constructor of Shapes from Parts—Looks Like

Uses manipulatives representing parts of shapes, such as sides, to make a shape that “looks like” a goal shape. May think of angles as a corner (which is “pointy”).

TSG Objectives

21.b.7	Explores and describes spatial relationships and shapes	Understands shapes	
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Shapes: Corner (Angle) Recognizer

Recognizes angles as separate geometric objects, at least in the limited context of “corners.”

TSG Objectives

21.b.10	Explores and describes spatial relationships and shapes	Understands shapes	Distinguishes essential attributes of triangles, rectangles, squares, trapezoids, half circles, and quarter circles; visualizes and creates known shapes
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Shapes: Parts of Shapes Identifier

Identifies shapes in terms of their components.

TSG Objectives

21.b.12	Explores and describes spatial relationships and shapes	Understands shapes	Uses essential attributes to label and create quadrilaterals, pentagons, hexagons, and cubes; visualizes and predicts the results of combining and taking apart two-dimensional and three-dimensional shapes
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Trajectory Level

TSG Objectives

21.b.11 Explores and describes spatial relationships and shapes

Understands shapes

Shapes: Property Class Identifier Uses class membership for shapes (e.g., to sort or consider shapes “similar”) explicitly based on properties, including angle measure. Is aware of restrictions of transformations and also of the definitions and can integrate the two. Sorts hierarchically, based on properties.

TSG Objectives

21.b.14 Explores and describes spatial relationships and shapes

Understands shapes

Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions

Shapes: Shape Class Identifier Uses class membership (e.g., to sort), not explicitly based on properties.

TSG Objectives

21.b.14 Explores and describes spatial relationships and shapes

Understands shapes

Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions

TSG Objectives

21.b.13 Explores and describes spatial relationships and shapes

Understands shapes

Shapes: Shape Identifier Names most common shapes, including, for example, rhombuses, hexagons, octagons, and trapezoids, without making mistakes, such as calling ovals “circles.” Recognizes (at least) right angles, so distinguishes between a rectangle and a parallelogram without right angles.

Trajectory Level

TSG Objectives

21.b.10	Explores and describes spatial relationships and shapes	Understands shapes	Distinguishes essential attributes of triangles, rectangles, squares, trapezoids, half circles, and quarter circles; visualizes and creates known shapes
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Shapes: Shape Matcher- Identical, Orientations, Sizes

Identical - Matches familiar shapes (circle, square, typical triangle) with same size and orientation. Sizes - Matches familiar shapes with different sizes. Orientations - Matches familiar shapes with different orientations.

TSG Objectives

21.b.3	Explores and describes spatial relationships and shapes	Understands shapes	
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TSG Objectives

21.b.2	Explores and describes spatial relationships and shapes	Understands shapes	Matches two identical shapes
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Shapes: Shape Matcher—More Shapes, Sizes, Orientations, & Combinations

Matches a wider variety of shapes with same size and orientation. Matches a wider variety of shapes with different sizes and orientations. Matches combinations of shapes to each other.

TSG Objectives

21.b.5	Explores and describes spatial relationships and shapes	Understands shapes	
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Shapes: Shape Property Identifier

Uses properties explicitly. Can see the invariants in the changes of state or shape, but maintaining the shapes' properties.

Trajectory Level

TSG Objectives

21.b.14	Explores and describes spatial relationships and shapes	Understands shapes	Classifies known shapes into higher and subordinate categories; provides rationale for classifications; divides shapes into parts with equal areas and expresses the parts as unit fractions
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Shapes: Shape Recognizer-All Rectangles

Recognizes rectangles of all sizes, shapes, and orientations.

TSG Objectives

21.b.8	Explores and describes spatial relationships and shapes	Understands shapes	Shows that shapes remain the same when they are turned, flipped, or slid; breaks apart or combines shapes to create different shapes and sizes
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Shapes: Shape Recognizer—Circles, Squares, and Triangles

Recognizes some less typical squares and triangles and may recognize some rectangles, but usually not rhombuses (diamonds). Often doesn't differentiate sides/corners.

TSG Objectives

Shapes: Shape Recognizer—More Shapes

Recognizes most familiar shapes and typical examples of other shapes, such as hexagon, rhombus (diamond), and trapezoid.

TSG Objectives

Shapes: Shape Recognizer-Typical

Recognizes and names a typical circle, square, and, less often, triangle. May physically rotate shapes in atypical orientations to mentally match them to a prototype.

TSG Objectives

21.b.6	Explores and describes spatial relationships and shapes	Understands shapes	Describes basic 2- and 3-dimensional shapes by using own words; recognizes basic shapes when they are presented in a new orientation
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Trajectory Level

TSG Objectives

21.b.10	Explores and describes spatial relationships and shapes	Understands shapes	Distinguishes essential attributes of triangles, rectangles, squares, trapezoids, half circles, and quarter circles; visualizes and creates known shapes
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TSG Objectives

21.b.4	Explores and describes spatial relationships and shapes	Understands shapes	Identifies a few basic shapes (circle, square, triangle)
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Shapes: Side Recognizer Identifies sides as distinct geometric objects with attributes.

TSG Objectives

21.b.9	Explores and describes spatial relationships and shapes	Understands shapes	
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TSG Objectives

21.b.10	Explores and describes spatial relationships and shapes	Understands shapes	Distinguishes essential attributes of triangles, rectangles, squares, trapezoids, half circles, and quarter circles; visualizes and creates known shapes
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Trajectory Level

Spatial Orientation

Spatial Orientation: Coordinate Plotter

Reads and plots coordinates on maps.

TSG Objectives

21.a.8	Explores and describes spatial relationships and shapes	Understands spatial relationships	Uses and makes simple sketches, models, or pictorial maps to locate objects
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Spatial Orientation: Foundations of Spatial Orientation

Uses the earliest of two types of cognitive systems for spatial orientation—knowing where you are and how to get around in the world.

- 1. Response Learning:** Uses the first self-based system – that is, related to the child’s own position and movements. Notes a pattern of movements that have been associated with a goal.
- 2. Cue Learning:** Uses the first external-based systems, based on familiar landmarks.

TSG Objectives

Spatial Orientation: Framework User

Uses general frameworks that include the observer and landmarks. May not use precise measurement even when that would be helpful, unless guided to do so. Can follow and create maps, even if spatial relations are transformed.

TSG Objectives

21.a.9	Explores and describes spatial relationships and shapes	Understands spatial relationships	
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TSG Objectives

21.a.8	Explores and describes spatial relationships and shapes	Understands spatial relationships	Uses and makes simple sketches, models, or pictorial maps to locate objects
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Trajectory Level

Spatial Orientation: Local Framework User

Locates objects after moving, maintaining the overall shape of the arrangement of objects. Represents objects' positions relative to landmarks (e.g., about halfway in between two landmarks) and keeps track of own location in open areas or mazes. Uses spatial vocabulary to direct attention to spatial relations. Uses coordinate labels in simple situations such as games.

TSG Objectives

21.a.6	Explores and describes spatial relationships and shapes	Understands spatial relationships Uses and responds appropriately to positional words indicating location, direction, and distance
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Spatial Orientation: Local-Self Framework User

Uses distant landmarks to find objects or location near them, even after they have moved themselves relative to the landmarks, if the target object is specified ahead of time. Orients a horizontal or vertical line in space (Rosser, Horan, Mattson, & Mazzeo, 1984). Uses spatial vocabulary to direct attention to spatial relations, including more difficult terms such as "beside" and "between."

TSG Objectives

21.a.5	Explores and describes spatial relationships and shapes	Understands spatial relationships
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TSG Objectives

21.a.4	Explores and describes spatial relationships and shapes	Understands spatial relationships Follows simple directions related to proximity (beside, between, next to)
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Trajectory Level

TSG Objectives

21.a.3 Explores and describes spatial relationships and shapes Understands spatial relationships

TSG Objectives

21.a.2 Explores and describes spatial relationships and shapes Understands spatial relationships Follows simple directions related to position (in, on, under, up, down)

Spatial Orientation: Map User

Locates objects using maps with pictorial cues. Extrapolates (extends) two coordinates, understanding the integration of them to one position, as well as use coordinate labels in simple situations.

TSG Objectives

21.a.7 Explores and describes spatial relationships and shapes Understands spatial relationships

TSG Objectives

21.a.8 Explores and describes spatial relationships and shapes Understands spatial relationships Uses and makes simple sketches, models, or pictorial maps to locate objects

Spatial Orientation: Path Integrater

Remembers and can repeat movements they have made including the approximate distances and directions.

TSG Objectives



Trajectory Level

Spatial Orientation: Place Learner

Creates “mental maps” by storing locations, distances, and directions to landmarks and solves spatial problems. Uses the walls of a room as a frame of reference; uses spatial vocabulary, such as "in," "on," and "under," along with vertical directionality terms as "up" and "down."

TSG Objectives

Spatial Orientation: Route Map Follower

Follows a simple route map, with more accurate direction and distances.

TSG Objectives

21.a.8	Explores and describes spatial relationships and shapes	Understands spatial relationships Uses and makes simple sketches, models, or pictorial maps to locate objects
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Spatial Orientation: Small Local Framework User

Locates objects after movement, even if target is not specified ahead of time. Searches a small area comprehensively, often using a circular search pattern. Uses words referring to frames of reference such as "in front of" and "behind" or "left" and "right." In meaningful graphing contexts, extrapolates lines from positions on both axes (like a coordinate grid) and determines where they intersect.

TSG Objectives

21.a.6	Explores and describes spatial relationships and shapes	Understands spatial relationships Uses and responds appropriately to positional words indicating location, direction, and distance
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Trajectory Level

Spatial Visualization

Spatial Visualization: Beginning Slider, Flipper, Turner

Uses the correct motions guided by more developed intuition, but not always accurate in direction and amount (adjusts these with trial and error). Knows a shape has to be flipped to match another shape, but flips it in the wrong direction.

TSG Objectives

Spatial Visualization: Concrete Slider, Flipper, Turner

Can move shapes to a location by physical trial and error.

TSG Objectives

Spatial Visualization: Diagonal Mover

Performs diagonal slides and flips as well as all motions from previous levels. Knows a shape must be turned flipped over an oblique line (45° orientation) to fit into a puzzle.

TSG Objectives

Spatial Visualization: Intuitive Mover: Foundations

Explores the size and shape of objects by observing them as they move in space, using trial and error to discover how they fit into space, and eventually predicting what will fit inside a space without attempting all possible solutions. Such skills will eventually support future spatial visualization.

TSG Objectives

Spatial Visualization: Mental Mover

Predicts results of moving shapes using mental images (any direction or amount).

TSG Objectives

Spatial Visualization: Simple Slider and Turner

Slides and turns objects accurately in easy tasks, guided by an early intuition that starts the motion and then adjusts (the motion, direction, or amount) in real time as the motion is carried out.

Trajectory Level

TSG Objectives

Spatial Visualization: Slider, Flipper, Turner

Performs slides and flips, often only horizontal and vertical, using manipulatives but guided by mental images of these motions (of turns of 45, 90, and 180° and flips over vertical and horizontal lines). That is, they can mentally imagine the motion and the result of it. Knows a shape must be turned 90° to the right to fit into a puzzle.

TSG Objectives

Trajectory Level

Subitizing

Subitizing: Conceptual Subitizer to 10

Verbally labels most briefly shown arrangements of all numbers 2 to 10. Children may know some familiar ones (“5 and 5 make 10” is common) early, but this level is reached when most all combinations of all numbers up to 10 are recognized (e.g., 7 and 2 seen as 9; 5 and 3 seen as 8; etc.). Uses structures such as tens-frames to recognize larger quantities.

TSG Objectives

20.f.5	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations
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TSG Objectives

20.b.6	Uses number concepts and operations	Quantifies	Makes sets of 6-10 objects and then describes the parts; identifies which part has more, less, or the same (equal); counts all or counts on to find out how many
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TSG Objectives

20.f.4	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 10 using mental strategies (counting on, making ten, decomposing/recomposing, addition/subtraction relationship, and easier equivalent known sums)
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Subitizing: Conceptual Subitizer to 20

Verbally labels structured arrangements of 10 to 20, shown only briefly, by seeing the parts and quickly knowing the whole. Spontaneously makes use of a top-down strategy to subitizing large quantities (Nes, 2009). Children may know some familiar ones (“10 and 10 make 20” is common) early, but this level is reached when most all combinations of numbers from 1 to 10 are recognized (e.g., 7 and 9 is seen as 16).

TSG Objectives

20.b.8	Uses number concepts and operations	Quantifies	Solves simple equal share problems; makes sets of 11 – 20 objects and then describes then parts
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Trajectory Level

TSG Objectives

20.b.9 Uses number concepts and operations Quantifies

TSG Objectives

20.b.7 Uses number concepts and operations Quantifies

TSG Objectives

20.f.6 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts whole numbers fluently within 20 using previously learned mental strategies; knows all the addition combinations of two, one-digit numbers from memory

Subitizing: Conceptual Subitizer to 5

Verbally labels all arrangements to about 5, shown only briefly, by seeing the parts and quickly knowing the whole. Conceptual subitizing refers to the ability of children to identify a whole quantity as a result of composing smaller quantities (recognized through perceptual subitizing) that make up the whole.

TSG Objectives

20.b.5 Uses number concepts and operations Quantifies

TSG Objectives

20.f.3 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations

TSG Objectives

20.f.2 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts whole numbers fluently within five

TSG Objectives

20.b.4 Uses number concepts and operations Quantifies Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts

Learning Trajectories: Alignment to Teaching Strategies GOLD



Trajectory Level

Subitizing: Conceptual Subitizer to 7

Verbally labels all arrangements to 6, then 7, when shown only briefly.

TSG Objectives

Subitizing: Conceptual Subitizer with Place Value

Verbally labels structured arrangements, shown only briefly, using groups, skip counting, and place value.

TSG Objectives

20.b.12	Uses number concepts and operations	Quantifies	Answers how much questions about wholes portioned into equal size shares of two and four; verbally labels each part and describes its relationship to the whole
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TSG Objectives

20.f.7	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	
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TSG Objectives

20.b.11	Uses number concepts and operations	Quantifies	
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TSG Objectives

20.b.10	Uses number concepts and operations	Quantifies	Answers how much questions about wholes portioned into equal size shares of two and four; verbally labels each part and describes its relationship to the whole
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TSG Objectives

20.f.8	Uses number concepts and operations	Applies number combinations and mental number strategies in mathematical operations	Adds and subtracts while numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on
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Subitizing: Conceptual Subitizer with Place Value and Multiplicative Thinking

Verbally labels structured arrangements, shown only briefly, using groups, multiplicative thinking, and place value. This level builds on the previous level, such that children are able to use the base-10 system to conceptually subitize larger numbers. Children are able to verbalize the quantity of 10's they see.

Trajectory Level

TSG Objectives

20.b.13 Uses number concepts and operations Quantifies

TSG Objectives

20.b.14 Uses number concepts and operations Quantifies Compares fractions and explains them using physical models, pictorial representations, and number lines

TSG Objectives

20.b.15 Uses number concepts and operations Quantifies

TSG Objectives

20.f.9 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations

TSG Objectives

20.f.8 Uses number concepts and operations Applies number combinations and mental number strategies in mathematical operations Adds and subtracts whole numbers fluently within 1,000; multiplies and divides whole numbers fluently within 100 using previously learned mental strategies, the relationships between addition/subtraction and multiplication/division, and algorithms based on

Subitizing: Maker of Small Collections

Makes a small collection (usually 1–2 and possibly 3) with the same number as another collection (via mental model; i.e., not necessarily by matching—for that process, see Compare Number). Might also be verbal but often is not. May not recognize spatial structures at first, and may count this.

TSG Objectives

20.b.3 Uses number concepts and operations Quantifies

Trajectory Level

TSG Objectives

20.b.2	Uses number concepts and operations	Quantifies	Demonstrates understanding of the concepts of one, two, and more
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Subitizing: Number Senser

Has inborn specific "sensors" for number from the first months of life without explicit knowledge of number. Intuitively distinguishes between groups of 1 and of 2 (and possibly 2 and 3). Also shows sensitivity to ratios of quite large numbers. (Approximate Number System, or ANS). These are pre-math, foundational abilities.

TSG Objectives

20.b.1	Uses number concepts and operations	Quantifies
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Subitizing: Perceptual Subitizer to 4

Instantly recognizes collections up to 4 briefly shown and verbally names the number of items.

TSG Objectives

20.b.4	Uses number concepts and operations	Quantifies	Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts
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Subitizing: Perceptual Subitizer to 5

Instantly recognizes briefly shown collections up to 5 and verbally names the number of items. Recognizes and uses spatial and numeric structures beyond the situations in which they were already experienced (i.e., in which they were initially learned).

TSG Objectives

20.b.4	Uses number concepts and operations	Quantifies	Recognizes and names the number of items in a small set (up to five) instantly; combines and separates up to five objects and describes the parts
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Subitizing: Small Collection Namer

Names groups of 1, 2, and 3 with increasing accuracy. Most children of about 34-39 months of age can accurately name groups of 1, 2, and 3. Many children learn to recognize and name groups of 4 about 6 months later. For a Maker of Small Collections (the previous level), the child may rely on matching strategies to make their small collection. In Small Collection Namer, the child is actually able to recognize small groups without relying on a model or matching strategy.

Trajectory Level

TSG Objectives

20.b.2	Uses number concepts and operations	Quantifies	Demonstrates understanding of the concepts of one, two, and more
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Subitizing: Very Small Number Recognizer

Begins connecting small quantities to number words to form an explicit idea of cardinality, or “how-many-ness.” Following the child’s first birthday, the number words “one” and “two” are often learned. Other general terms such as “more” and “less” usually follow. Only over time do they begin to understand that all groups labelled with the same number word have the same amount.

TSG Objectives

20.b.1	Uses number concepts and operations	Quantifies
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Trajectory Level

Volume

Volume: 3-D Array Structurer

Has an abstract understanding of the rectangular prism volume formula. Shows a propensity for multiplicative comparisons, coordinates multiplicative and additive comparisons flexibly. With linear measures or other similar indications of the three dimensions, multiplicatively iterates cubes in a row, column, and/or layers to determine the area. Constructions and drawings are not necessary. In multiple contexts, children can compute the volume of rectangular prisms from its dimensions and explain how that multiplication creates a measure of volume.

TSG Objectives

22.a.14	Compares and measures	Measures objects	Solves one-step word problems related to measurement of liquid volume, mass, area, and perimeter.
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TSG Objectives

22.a.15	Compares and measures	Measures objects
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Volume: 3-D Row and Column Structurer

Able to coordinate flexibly filling, packing, building aspects of volume. Shows a propensity for additive comparisons (e.g., “this one has 12 more”) but may show some nascent multiplicative comparisons (e.g., “this one is four times as big”). Initially counts or computes (e.g., number of rows times number of columns) the number of cubes in one layer, and then uses addition or skip counting by layers to determine the total volume. Eventually moves to multiplication (e.g., number of cubes in a layer times number of layers).

TSG Objectives

22.a.13	Compares and measures	Measures objects
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TSG Objectives

22.a.14	Compares and measures	Measures objects	Solves one-step word problems related to measurement of liquid volume, mass, area, and perimeter.
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Trajectory Level

Volume: Initial Composite 3-D Structurer

Understands cubes as filling a space but does not use layers or multiplicative thinking. Moves to more accurate counting strategies. Relates number of cubes to cubic units as measured by capacity. Given a graduated cylinder marked in cubic-inch units, child understands that sand filled to the 10 in the cylinder would fill a box that holds ten, 1-inch cubes. Begins to visualize and operate on composite units such as rows or columns (what we call a 1x1xn core). Iterates to pack the space completely, accounting for “internal/ hidden” cubes. Decomposes space, allowing for accurate use of units and subunits. Recognizes when a box is half full, visualizes remaining rows or columns.

TSG Objectives

22.a.13 Compares and Measures objects
measures

TSG Objectives

22.a.12 Compares and Measures objects Measures and compares the length of two objects using
measures standard length units

Volume: Volume Filler

Can compare two containers by pouring one into the other (although can be confused at “which holds more” at first). Fills a container using another (smaller container) and counts the number needed to completely fill the larger container (but may not use accurately filled scoops and may not focus on quantifying the total volume or capacity). In packing situations, places cubes into a rectangular box to fill it. Eventually packs entire box with cubes in an organized way. Compares objects by physically or mentally aligning; refers to at least two dimensions of objects. May be able to compare two containers using a third container and transitive reasoning.

TSG Objectives

22.a.4 Compares and Measures objects Compares and orders a small set of objects as appropriate
measures according to size, length, weight, area, or volume.

TSG Objectives

22.a.3 Compares and Measures objects
measures



Trajectory Level

Volume: Volume Quantifier

Partial understanding of cubes as filling a space. Able to estimate number of scoops needed to fill. Able to attend to both the portion of container filled and the portion remaining unfilled. Recognizes when container is half full. Exhibits initial spatial structuring. Packs box neatly and completely with cubes; may count one cube at a time, while packing, to determine total. Compares objects by physically or mentally aligning and explicitly recognizing three dimensions.

TSG Objectives

22.a.8	Compares and measures	Measures objects	Uses measurement words and some standard measurement tools accurately; uses ordinal numbers from first to tenth
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TSG Objectives

22.a.6	Compares and measures	Measures objects	Uses multiples of the same unit to measure; uses numbers to compare; knows the purpose of standard measuring tools
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TSG Objectives

22.a.7	Compares and measures	Measures objects	
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Volume: Volume Quantity Recognizer

Identifies capacity or volume as attribute. Builds with blocks, associating more blocks with terms like “big” and fewer blocks with terms like “small.”

TSG Objectives

22.a.3	Compares and measures	Measures objects	
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TSG Objectives

22.a.2	Compares and measures	Measures objects	Makes simple comparisons between two objects
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Volume: Volume Senser: Foundations

Sensitive to volume even in the first year; however, they may not for some time explicitly recognize volume as an attribute (separate from general size, such as “small” and “big”).

TSG Objectives

Trajectory Level

Volume: Volume Unit Relater and Repeater

Uses simple units to fill containers, with accurate counting. Relates size and number of units explicitly; understands that fewer larger than smaller units will be needed to fill or pack a given container. Can accurately convert units in 1:2 ratio.

TSG Objectives

22.a.11 Compares and Measures objects
measures

TSG Objectives

22.a.9 Compares and Measures objects
measures

TSG Objectives

22.a.10 Compares and Measures objects Measures length accurately and expresses the measurement in
measures whole numbers

TSG Objectives

22.a.11 Compares and Measures objects
measures

TSG Objectives

22.a.12 Compares and Measures objects Measures and compares the length of two objects using
measures standard length units

TSG Objectives

22.a.13 Compares and Measures objects
measures

TSG Objectives

22.a.12 Compares and Measures objects Measures and compares the length of two objects using
measures standard length units