Early Childhood Corner

The Earliest Geometry

At the core of mathematics in the early years are the Number and Geometry Standards. (NCTM 2000, p. 77)

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In keeping with the early childhood chapter of *Principles and Standards for School Mathematics,* this department examines activities and children's thinking in geometry and, in the next issue, number. From prekindergarten to grade 12, the Geometry Standard addresses four main areas: properties of shapes, location and spatial relationships, transformations and symmetry, and visualization. For each area, we quote the goal of the Standard and the associated early-childhood expectations. We then present snippets of research and sample activities to develop ideas within each area with students.

Shapes and Their Properties

The NCTM's *Standards* document states that instructional programs for grades pre-K–12 should enable all children to "analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships" (p. 96). The expectations for the early years are that children should—

- recognize, name, build, draw, compare, and sort two- and three-dimensional shapes;
- describe attributes and parts of two- and three-dimensional shapes; and
- investigate and predict the results of putting shapes together and taking them apart.

Knowledge of shapes in young children

Children begin forming concepts of shape long before they enter school. They may first learn to recognize shapes by their overall appearance, stating, for example, that a given figure is a rectangle because "it looks like a door." Or they might focus on one aspect of a shape, for instance, calling a figure a triangle because it is "sharp." Children perform well using such thinking. For example, they accurately identify circles and squares, even with "tricky" distractors. They do not do as well with rectangles and triangles but still identify 50 to 60 percent correctly. Even so, they often believe that squares that are not placed horizontally are no longer squares.

Do teachers need to address these ideas with very young children? Yes! Young children's ideas about shapes—including limited ideas—stabilize as early as six years old. If a six-year-old believes, for example, that nonisosceles triangles are not triangles, he or she will likely continue to believe that misconception for years to come—regardless of what the teacher or a textbook says.

Prepared by the editors, **Douglas Clements,** *clements@acsu.buffalo.edu, and* **Julie Sarama,** *jsarama@buffalo.edu, State University of New York at Buffalo, Buffalo, NY 14260.* *This section addresses the early childhood teacher's need to support young children's emerging mathematics understandings and skills in a context that conforms with current knowledge about the way that children in prekindergarten and kindergarten learn mathematics. Readers are encouraged to send manuscripts for this section to "Early Childhood Corner," NCTM, 1906 Association Drive, Reston, VA 20191-9988.*

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Further, even very young children *can* learn to name and sort shapes accurately and describe their parts and attributes. One of the youngest subjects in our research, a three-year-old, scored higher on such a task than every six-year-old! Such children have had rich experiences with varied shapes and have talked about their attributes. What types of experiences might teachers provide?

Geometry every day

One activity that encourages children to describe attributes of shapes involves using fabric paint or masking tape to make a shape on the rug. The children are asked if they want to stand inside or outside the shape to sing a song. To get permission to stand where they choose, they must answer a question, such as "How do you know the shape is a triangle?" To answer correctly, the children must describe the shape's sides and angles. A similar activity uses shapes taped to the classroom tables. Students are called to line up by describing the shape that is taped to their table. The shapes are changed weekly.

Getting into shape

After identifying and talking about several three-dimensional shapes, have children pretend that they are inside one of the shapes. For example, after displaying an oatmeal box, say to the children, "Pretend that you are inside a cylinder like this. Your fingers are touching the inside of your cylinder. Close your eyes, and touch your cylinder. Now touch the top of the cylinder. Go around the top with your hands. What shape do you trace? Go around the bottom of your cylinder with your foot. What shape do you trace? Move all around inside your cylinder."

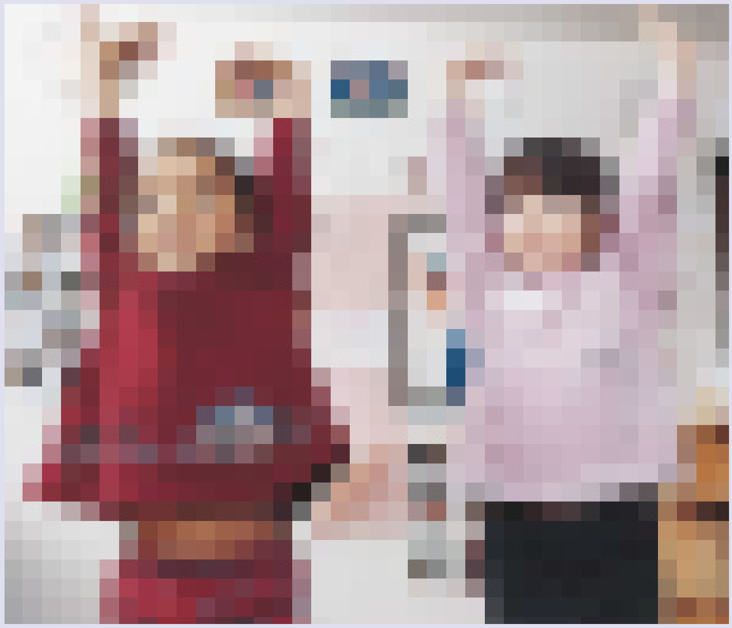
This activity can be repeated with other shapes, such as a cube (see **fig. 1**). Children can "feel" each of the faces and describe one in detail. What shape is the face? Ask the children to feel the opposite faces, first the top and bottom, then opposite sides. How many faces does the cube have? Ask whether they can move their hands to a corner of the cube. How many corners can they find at the top? How many at the bottom? How many corners does the cube have in all? On another day, have children actually get into a large box or tube. They can feel the outsides and insides of the box and talk about what they are touching.

Feeling good about shapes

Gather two identical groups of shapes. Show one group to the class, and hide the other. Pass around the displayed shapes. Encourage children to run their fingers around each shape. Then secretly put one shape from the hidden group into the "feely box" **(fig. 2).** Have children feel the shape and try to guess which of the displayed shapes they are feeling. Children love to repeat this game many times and can be invited to play it on their own. Use shapes that emphasize two dimensions and, on other days, three-dimensional solids. Have children match shapes to pictures or describe the shapes that they are feeling so that friends can choose the matching displayed shape. If children have difficulty, start with real-world objects. To assess children's understanding, ask such questions as "How did you know? What part did you feel? Show me how you felt it."

FIGURE 1

Children, pretending that they are inside a giant cube, carefully "feel" the sides and top.



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Location and Spatial Relationships

The *Standards* document states that pre-K–12 students should be able to "[s]pecify locations and describe spatial relationships using coordinate geometry and other representational systems" (p. 96). Young children should—

- describe, name, interpret, and apply ideas of relative position in space;
- describe, name, interpret, and apply ideas of direction and distance in navigating space; and
- find and name locations with simple relationships, such as "near to," and use coordinate systems, such as those in maps.

Unit blocks and spatial relations

High-quality early childhood classrooms are filled with geometric manipulatives and opportunities to learn spatial ideas, although we do not always recognize them. The inventor of today's unit blocks, Caroline Pratt, tells a story of children who were trying to make a stable with enough room for a toy horse to fit inside. The teacher told one of the children, Diana, that she could have the horse when she had made a stable for it. Diana and Elizabeth began to build a small structure, but the horse did not fit. Diana had made a large stable but with a low roof. After several unsuccessful attempts to get the horse inside, she removed the roof, added blocks to the walls to make the roof higher, and replaced the roof. She then tried to put what she had done into words: "Roof too small." The teacher introduced new words, "high" and "low," to Diana, and she gave a new explanation to the other children.

Children's work with blocks gives them experience with perspective

Maps

Remarkably, even preschoolers can work profitably with maps. Many three-year-olds can build and understand simple maps using landscape toys, such as houses, cars, and trees. Other children may place such objects randomly. Children produce quite different maps because they have different drawing and map-building skills. These skills can develop with experience, although cognitive and physical development also play a role.

Most children can also learn *from* maps. For example, preschoolers and kindergartners can learn a route through a large playhouse more quickly if they examine a map beforehand. Young children have much to learn about maps, of course. For example, preschoolers can recognize roads on a map but may suggest that the tennis courts on a map are doors!

Children's work with blocks gives them experience with perspective. For example, they might identify block structures from various viewpoints, matching views of the same structure portrayed from different perspectives. They may also try to find the viewpoint from which a photograph was taken. Such experiences address preschoolers' confusion of perspective when they "see" windows and doors of buildings in vertical aerial photographs.

Similarly, children need to develop ideas about direction. They might mark a path from a table to the wastebasket with masking tape, then, with the teacher, draw a map of this path. Items that appear alongside the path, such as a table or an easel, can be added to the map.

Location is another important mathematical idea. Children might arrange cutout shapes of a tree, swing set, and sandbox on a felt board to make a simple map of the playground. They can discuss the idea that moving an item, such as a table, in the schoolyard would change the map. Using the map, locate children who are sitting in or near the tree, swing set, and sandbox. Plan playground scavenger hunts in which students give and follow directions or clues.

FIGURE 2

A student uses the "feely box."

(a)
This child is trying to match the block inside the box with one of those on the table.

(b)
He takes it out to check.

(c)
To make a good "feely box," start with a sturdy cardboard box; cut two holes on opposite sides; and around each hole, firmly attach a tube sock with the toes cut off, making a "tunnel" that is open on both ends. This device allows children to get both hands into the box without accidentally (or otherwise!) peeking. You can substitute a cloth or paper bag for the box.

Moving and mapping on computers

Computer activities can facilitate children's learning of navigational and map skills. Young children can abstract and generalize directions and measurement by working with navigation environments that require them to direct a car or an animal around the screen. One kindergartner abstracted the geometric notion of *path* by saying, "A path is like the trail a bug leaves after it walks through purple paint." Simple coordinate games on computers can help children learn location ideas. For example, the on-screen version of Battleship requires players to guess a location by given coordinates, such as "B, 5." When children enter a coordinate to move an object but it goes to a location that is different from what they had planned, the feedback is natural and meaningful.

FIGURE 3

Children reflect on geometric motions by manipulating a physical shape and relating the movement to the shape on the screen.

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Transformations and Symmetry

The *Standards* document states that pre-K–12 students should "[a]pply transformations and use symmetry to analyze mathematical situations" (p. 96). Young children should—

- "recognize and apply slides, flips, and turns"; and
- "recognize and create shapes that have symmetry."

Geometric motions

Children use geometric motions intuitively when they solve puzzles. They turn the pieces, flip them over, and slide them into place. If they use computer programs to solve puzzles, they must choose each motion deliberately (see **fig. 3**). Such activities help students become aware of the motions and the result of each one. They also learn that changing an object's position or orientation does not change its size or shape.

Symmetry

Many activities help children explore symmetry. Children's unit-block buildings frequently display symmetry (see **fig. 4**). One teacher read and explored *Make a Bigger Puddle, Make a Smaller Worm* (Walter 1971) with her class. She then had the children make symmetric shapes using mirrors and by folding and cutting paper. Children can also explore symmetry by using computers. Activities that ask children to complete the other "half" of a symmetric design or explore pattern-block designs with built-in mirrors allow children to explore symmetry dynamically (see **fig. 5**). The design explored in such activities is always symmetric.

FIGURE 4

Two students cooperate to create a symmetric building.

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Visualization

The *Standards* document recommends that pre-K–12 students "[u]se visualization, spatial reasoning, and geometric modeling to solve problems" (p. 96). Young children should—

- form mental images of geometric shapes by using spatial memory and spatial visualization;
- recognize and represent objects from different points of view;
- relate geometric ideas to number and measurement ideas; and
- recognize and locate geometric shapes and structures in the environment.

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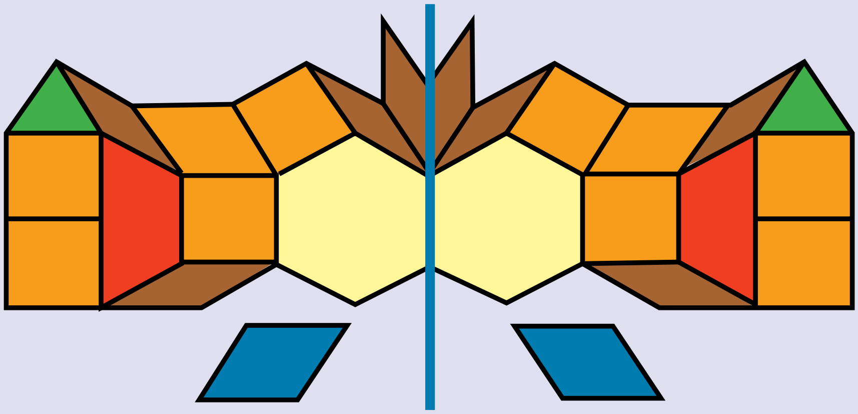
FIGURE 5

When a child creates or moves any shape, a corresponding "mirror shape" moves in a symmetrical fashion. Sliding one shape to the right causes its mirror image to slide simultaneously to the left. Turning a shape to the right causes its mirror image to rotate to the left by the same amount at the same time.



Visualizing shapes

Even young children can improve their ability to visualize geometric shapes and transformations. Children can be asked to close their eyes and think of a square: "What do you see? Now open your eyes, and look around the classroom to find that square you were thinking about. Who can tell me where a square is?" When children are comfortable with this activity, increase the complexity of your questions. For example, "I'm thinking of a figure that has only three straight sides and three corners. What is the shape that I am thinking about?" "Close your eyes. Picture a square sitting on its side. Now cut it in half from the bottom to the top. What shapes do you see?"

Quick images

Children enjoy quick-image activities, which build visual abilities. The basic idea is to show a shape or design for two seconds, then hide it. Ask children to make a copy of the shape. For example, show a simple design with manipulatives, and challenge children to reproduce it with their own manipulatives. Show a drawing, and ask the children to draw a copy. For each variation, encourage children to talk about what they see.

We should not underestimate the ideas that children can learn. The Agam program, for example, uses quick-image activities with three- to five-year-olds to teach ideas that sound quite abstract and mathematical, such as *horizontal lines* (Eylon and Rosenfeld 1990). Children view drawings of horizontal lines in various combinations for a few seconds, then reproduce them using sticks or by drawing. Later, they quickly look at objects in the room, such as a desk, and draw only the horizontal lines that they saw.

Shaping Up Mathematics with Geometry

Activities such as these help children learn more than geometry. The Agam program—designed to develop the "visual language" of children ages three to seven years—is one example. The children who used this program gained in geometric and spatial skills and in arithmetic and writing readiness. Our own current research supports these findings (Clements 1999). Early childhood education should address systematic, long-term instruction in geometry. Children are better prepared for all school tasks when they gain the thinking tools and representational competence of geometric and spatial sense.

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