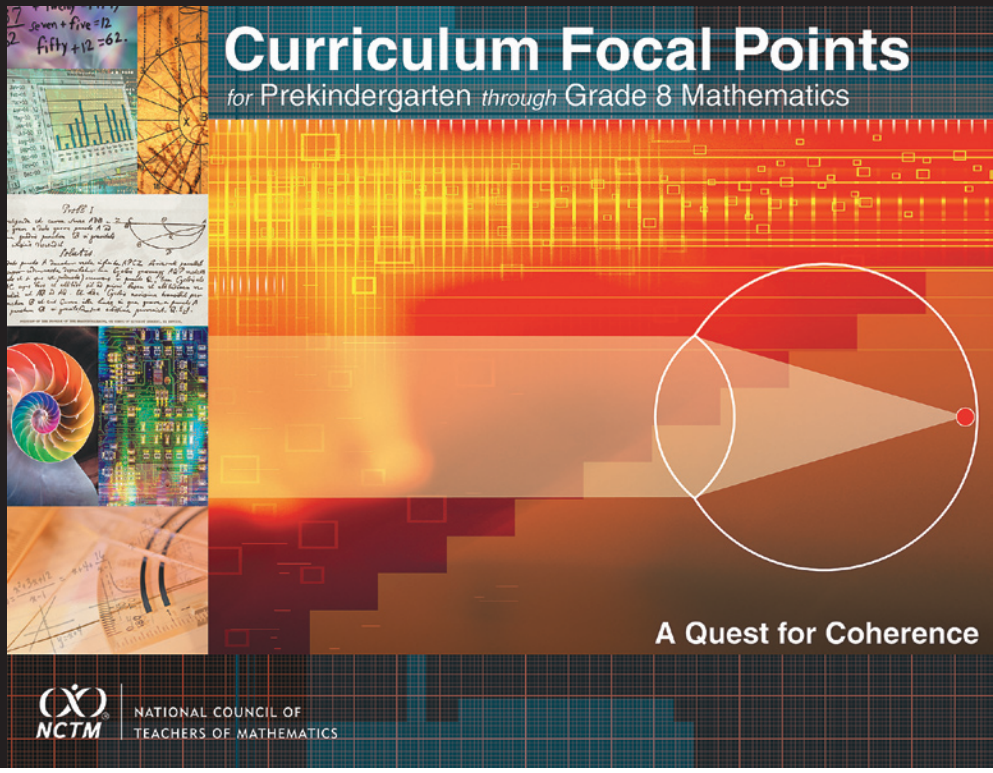
FOCAL POINTS—

Pre-K to Kindergarten



This is the second in a series of articles exploring the use of the National Council of Teachers of Mathematics' (NCTM's) 2006 publication, Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence. *The series introduction by NCTM President Skip Fennell, explaining what Curriculum Focal Points are and why NCTM developed them, appeared in the December 2007/January 2008 issue of* Teaching Children Mathematics *(page 315). In this and subsequent* TCM *articles, the authors of the various grade bands discuss the Focal Points for one or two grade levels. Because one principle of* Curriculum Focal Points *is that of cohesive curriculum, in which ideas develop across the grades, we encourage teachers of all grade levels to read the full series.*

"You have more. 1, 2, 3, 4, 5. I only have 4. Give me one."

"That won't help." Then *you'll* have 5, and *I'll* only have 4!"

Given the opportunity, four- and five-year-olds can surprise us with their mathematical knowledge and reasoning. As delightful as such a surprise is, in light of their age and the importance of social-emotional development, is it really important for all children at prekindergarten and kindergarten levels to have core knowledge? We think so, for several reasons.

Why Early Mathematics Is Critical

The early years are essential to children's development of mathematical competence. Consider two research results. First, mathematics is a core component of cognition. One extensive study showed that the better children were at reading as preschoolers, the better they were at reading in elementary school. And the better preschoolers were at mathematics, the better they were at mathematics in later grades. This is important information but not surprising. However, this is only half of the story. Early reading predicts only later *reading* ability. Mathematics in preschool predicts not only mathematics but also later literacy and reading (Duncan, Claessens, and Engel 2004). Mathematics is a general cognitive skill.

A second, related reason that early mathematics is important involves equity concerns. Gaps in early mathematics knowledge between income groups and between nations may be as wide at three and four years of age as at the elementary level. Mathematics knowledge in children from low-income families falls .55 standard deviations below that of middle-income families and a startling 1.24 standard deviations below that of high-income families. Curricula and programs using research-based developmental progressions of mathematical concepts and skills close these gaps; in some cases, lower-income children outperform their middle-class counterparts (Griffin 2004; Barrett et al. 2006; Clements and Sarama 2007c). Gaps in mathematics knowledge between income groups are significant, and good mathematics curricula and teaching can close these gaps.

By Douglas H. Clements and Julie Sarama

Douglas H. Clements, clements@buffalo.edu, and Julie Sarama, jsarama@buffalo.edu, are faculty members at the University of Buffalo, State University of New York, Buffalo, NY 14260.

Figure 1

Children at this level of counting skill can maintain a correspondence between their verbal counting and the stars.

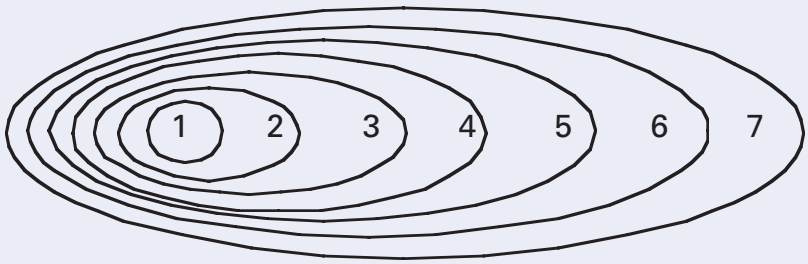
★ ★ ★ ★
"1, 2, 3, 4"

Figure 2

(a) Children at this level maintain correspondence *and* recognize that the last number answers the "how many?" question.

★ ★ ★ ★
"1, 2, 3, 4... *four!*"

(b) Children can learn that each successive number includes all the previous numbers within it.



What Mathematics?

What mathematics is most important to teach? The National Council of Teachers of Mathematics (NCTM) has released a new lens through which to view the content presented in *Principles and Standards for School Mathematics* (NCTM 2000). This companion publication, *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence* (NCTM 2006), is a major step toward defining the mathematics that every child should know and be able to do for each specific age or grade level.

The *Curriculum Focal Points* lens can help us view curriculum from two perspectives: the *concept* of a Focal Point and the *content* for a particular age or grade level (Schielack and Seeley 2007). From a concept perspective, the Focal Points help us reorganize what often have been long, perplexing lists of specific skills that educational systems at state, provincial, regional, and local levels require for each grade. The Focal Points help us see important mathematical ideas as interconnected packages of related concepts and skills. Further, instead of presenting lists, Focal Points concepts help us think about the developmental sequence of mathematical topics, no matter the age at which they are taught. As we shall see, within and across levels, such developmental sequences help us teach more effectively. Teachers can build on ideas in ways that respect both the logic of mathematics and children's mathematical thinking.

From a content perspective, the Focal Points identify specific subject matter for particular ages or grade levels, putting mathematics curriculum into a cohesive whole. This perspective is valuable to the classroom teacher, who can focus on the most important topics, connect these topics to one another, and thus provide children with an integrated understanding of mathematics.

As an educator, you still may feel responsible for that mandated list, but these two perspectives—concept and content—can help you develop your students' mathematical thinking in ways that are more effective for learning any mathematics. The Focal Points encompass core ideas, skills, and the research-based developmental sequences in which those ideas and skills are best learned.

Before we turn to specific mathematical content for prekindergarten and kindergarten, remember that the introductory statement at each grade level in *Curriculum Focal Points* emphasizes the central role of mathematical processes in a coherent curriculum. Each introductory statement also reminds us that instructional design in mathematics must support the development of key mathematical ideas in ways that promote problem solving, reasoning, communicating, and making connections as well as designing and analyzing representations.

Prekindergarten Mathematics

Number plays a key role in mathematical content, but so do geometry and spatial sense. For example, research in Israel and the United States shows that early geometry work leads to higher mathematics achievement, higher writing readiness, and higher IQ scores in the primary grades (Razel and Eylon 1990; Clements and Sarama 2007b). Within these two domains, number and geometry, we build the foundation for early and later mathematics.

The first prekindergarten Curriculum Focal Point addresses number and operations: "Developing an understanding of whole number, including concepts of correspondence, counting, cardinality,

and comparison." This Focal Point rests on research showing that the development of whole-number concepts and skills begins with two methods of quantification that develop in parallel: (1) recognizing the number of very small sets and (2) counting. Assigning a number to a set of objects is the first and most basic mathematical algorithm.

This much is clear from the first reading of *Curriculum Focal Points*. What may not be as clear is how each Focal Point also suggests a developmental sequence of each of these methods of quantification. For example, counting follows a developmental sequence: Children begin to count by learning to recognize that number words refer to quantity and by learning those number words in order—verbal counting. Next, they learn to use one-to-one correspondence in counting objects (see **fig. 1**). Later, they develop an understanding of the "cardinal principle"—the idea that the last number word in counting tells you "how many" in the set of all objects counted (see **fig. 2a**). This is not simply a "rule" to follow. To fully understand the idea of counting, children must understand that each number includes those that came before. **Figure 2b** illustrates this idea. Finally, preschoolers can learn to use this knowledge to compare the number in two sets. That is, "I counted 6 circles and 5 squares, so there are more circles, because 6 comes after 5 when we count." Again, children must understand that each counting number is quantitatively one more than the one before (see **fig. 3**).

Recognizing very small sets without counting is often a neglected method of quantification. This is unfortunate, because it is most children's first method of quantification, and it supports the development of number sense, counting, and arithmetic. When done quickly, such number recognition is called *subitizing* (Clements 1999). When children count, subitizing the number in the set both encourages and reinforces understanding of the cardinal principle that the last number word is the same as the number the child recognizes. Subitizing also follows developmental sequence. Most obviously, children learn to recognize the number in larger sets; but they also learn to recognize different arrangements of objects (illustrated in order of difficulty in **fig. 4**).

The second prekindergarten Focal Point targets geometry, emphasizing that children at this level

Figure 3

Building an ordered set of connecting cubes can help children see that counting numbers create a "one more" pattern.

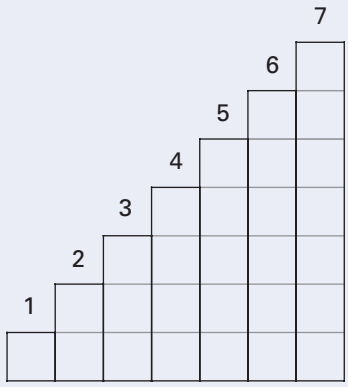


Figure 4

These arrangements show increasing levels of difficulty for subitizing a set of five objects.

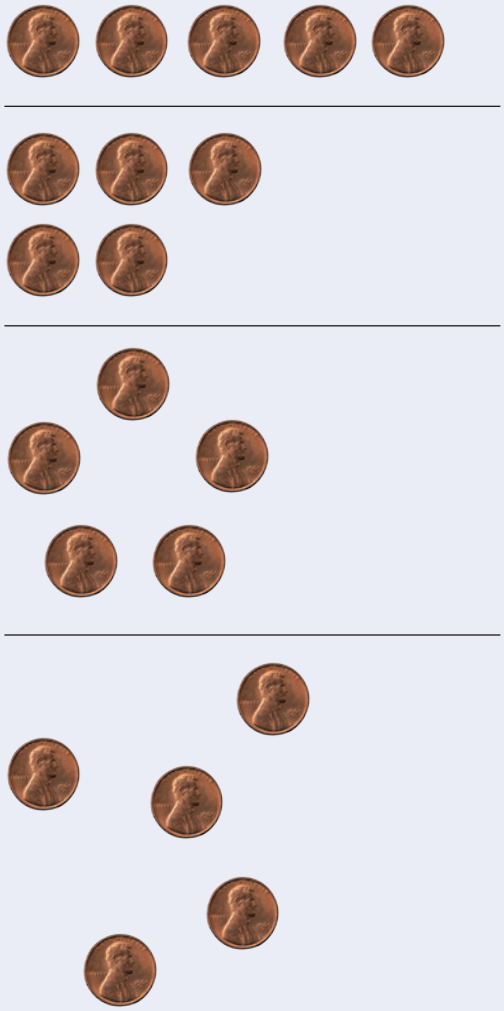
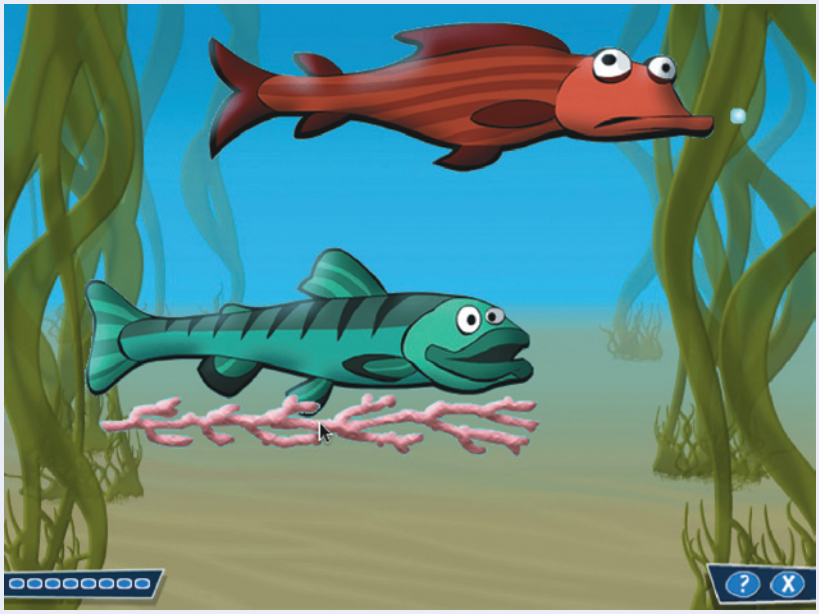


Figure 5

Children drag the coral to compare its length to the length of each of the two fish, determining which fish is longer (Clements and Sarama 2007a).



can observe and talk about shapes in the environment, which goes beyond naming a typical circle, square, or triangle. For example, they can learn to recognize different shapes of triangle—that is, not all are equilateral or isosceles—and to discuss these attributes. They can also develop a spatial sense, describe the relative positions of objects, and use important vocabulary such as *above* and *next to*. Such deep, extensive use of language obviously supports children's development of general cognition and literacy.

The third Focal Point for prekindergarten involves foundations for measurement. Children can identify measurable attributes and compare objects by using those attributes. They can recognize objects as *longer* or *shorter* and learn to differentiate whether *bigger* means length, area, weight, or some other attribute.

The prekindergarten Connections to the Focal Points are not merely other topics to cover. Just the opposite is true; they connect to the three main Focal Points, giving them additional depth and breadth. For instance, the Number and Operations Connection asks children to apply the knowledge from the three Focal Points in solving problems. For example, children might decide whether a given geometric figure is a triangle by checking that it is closed and by counting its three straight sides—reasoning it *must* be a triangle although it is "long and skinny." As another example, the Data Analysis Connection avoids having children create simple graphs (year after year!), instead building the foundations of data analysis by having children describe, sort, and compare physical and mathematical characteristics, such as size, quantity, or shape.

Kindergarten Mathematics

Kindergartners build on their foundations from prekindergarten. Of course, children who have not attended prekindergarten—or who, for any reason, have not developed these skills—need to work through the prekindergarten developmental sequences for each Focal Point. The Number and Operations Focal Point for kindergartners emphasizes that children use numbers, including written numerals, to represent quantities and to solve quantitative problems. Children might count the number of people at a table, produce a set with that number of pencils, and distribute that same number of pencils. In games or other activities, they can compare and order sets or numerals. Finally, using modeling and subitizing—counting-based strategies—they can solve joining and separating problems.

The Geometry Focal Point stresses that children at this level should use geometric ideas and vocabulary to observe their physical world. Children should learn to identify objects, shapes, orientations, and spatial relations, such as, "My block tower has six rectangular blocks with a triangle block on top." They can begin to reason about the attributes of these shapes: "All the rectangle blocks have horizontal tops and bottoms, so they stack. The triangle looks good at the top of the tower, and it has to be there because it doesn't have a horizontal face on its top." They should learn to identify, name, and describe a variety of shapes—squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids—presented in a variety of ways (e.g., with different sizes or orientations), as well as three-dimensional shapes, such as spheres, cubes, and cylinders. They should use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes.

In the Measurement Focal Point, kindergartners go beyond direct comparison of two objects to *ordering* objects by measurable attributes: They can line up dolls, or themselves, by height. They also compare the lengths of two objects indirectly by comparing both with a third object. For example, to determine if a desk will fit through a doorway, they

can compare the width of each by using the length of a stick. Although these skills clearly form the basis for understanding quantity and measurement, they also are foundational cognitive skills. Indeed, research indicates that mastering these skills is one of the strongest predictors of school success or failure (Clements and Sarama 2007b). Engaging in simple activities such as these (as well as the ones illustrated in **figs. 3** and **5**, Clements and Sarama 2007a) can close the gap in such cognitive skills.

As with prekindergarten, the kindergarten Connections to the Focal Points support the development of the main Focal Points and broaden children's mathematical knowledge. Children sort and re-sort by new attributes, extending their skill in classification, which is another foundational cognitive skill, as well as the basis of data analysis. They discuss simple navigational directions, such as, "Walk forward ten steps, turn right, and walk forward five steps," integrating their understanding of number, geometry, and measurement.

Next Steps

Download *Curriculum Focal Points* from NCTM. Consider how you can organize your teaching now to make use of the guidance in the book regarding what topics to stress and the developmental sequences that promote the best understanding of those topics. What unnecessary topics might you de-emphasize in your lessons or delete altogether? What ideas in number and geometry might you develop at deeper levels than in the past? How might you and your students better connect various topics? What groups, from grade-level groups to state and national organizations, might you work with to help your state, provincial, or school district standards and assessments align with NCTM's Curriculum Focal Points? Where do you want to focus your efforts?

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence is available at no charge for PDF download as a full document or by section at www.nctm.org/standards/focalpoints.aspx?id=282. The published document is available for sale through the NCTM catalog. Readers also may be interested in a similar series that began in the August 2007 issue of *Mathematics Teaching in the Middle School* and ran through the December 2007/January 2008 issue.

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